Systematics of nuclear properties in the framework of relativistic self-consistent models

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Fiz. Elem. Chastits At. Yadra **23**, 847–913 (July–August 1992)

Relativistic self-consistent theory of nuclear structure is considered. Such an approach has achieved a high degree of sophistication and in the framework of the Hartree and Hartree–Fock approximations describes quantitatively many properties of the ground states of finite nuclei: total binding energies of nuclei, charge-distribution radii, separation energies, and spin–orbit splittings. The Coulomb field of the nucleus is introduced naturally on the basis of vector dominance with allowance for the electromagnetic structure of the nucleons. The connection between the relativistic theory and the Hartree–Fock method with effective Skyrme forces is considered. The problems of the relativistic Dirac–Brueckner–Hartree–Fock theory for finite nuclei are discussed. The success of the relativistic Hartree and Hartree–Fock approximations ensure a good basis for the study of nuclear excitations in relativistic models.

Since the beginning of the seventies, there has been broad development in nuclear physics of the direction in which the nucleus is treated as a relativistic system; many new important results have been obtained in this field during the last 20 years.[1–7]

In particular, it has been established that the anomalously large spin–orbit coupling in nuclei, which is treated purely phenomenologically in the nonrelativistic description, can be explained naturally in the relativistic approach.

One of the advantages of the Dirac description compared with the Schrödinger picture is that in the framework of relativistic theory a natural explanation is provided for the energy dependence of the real part of the optical potential—at intermediate energies of the nucleons, the optical potential takes the characteristic shape of the "bottom of a wine bottle." This dependence was confirmed experimentally by investigations into the elastic scattering of protons by nuclei at intermediate energies.

The relativistic form of Kerman–McManus–Thaler theory has been widely developed. This approach does not contain free parameters. Relativistic calculations have been made in the framework of this theory for the differential cross section, analyzing power, and spin-flip function for elastic scattering of polarized protons with energy ~ 500 MeV by ^{40}Ca and ^{208}Pb nuclei. The calculations showed that the transition from the nonrelativistic description to the relativistic theory leads to a "dramatic" improvement of the agreement with experiment for polarization effects.

Already in the Hartree approximation the relativistic treatment makes it possible to obtain agreement with the experiments for the total binding energies of nuclei, the charge-distribution radii, and the separation energies. The binding energies and the charge radii are reproduced to an accuracy of a few percent. The density distributions of spherical doubly magic nuclei obtained in the relativistic approach are in good agreement with the electron scattering data. It has also been shown that saturation, a fundamental property of nuclei, is a relativistic kinematic effect, and self-consistent relativistic calculations have been made for both spherical and deformed nuclei.

Relativistic effects are most important for fields of anomalous parity [for which the multipolarity L determines the spatial parity by means of the relation $(-)^{L+1}$], i.e., for fields that couple the large and small components of the wave function. Such effects can be very important in investigations of the stability of nuclear ground states in external fields; fields whose symmetry differs from the symmetry of the ground state are of the greatest interest.

Relativistic effects in nuclei can also be important in the study of electromagnetic and β transitions, and also processes determined by the Coulomb interaction.

In addition, the relativistic approach, in particular, relativistic wave functions, are needed to describe characteristic phenomena in the region of intermediate energies, for example, (p, π^+) reactions.

An advantage of the approach to the description of nuclear properties considered in the present review is, in particular, that in this approach it is possible to give a unified description of not only the interaction of nucleons with nuclei but also that of other elementary particles: antiprotons, pions, hyperons, and kaons.

The development of a relativistic description of nuclei is inseparably connected with the successes of meson theory, in particular, the meson theory of nucleon–nucleon interactions.

During the last two decades, the meson approach to the description of nuclear systems developed especially strongly, and extensive reviews on the meson theory of nuclear forces have been published. Important successes have been achieved in the description of nuclear matter and finite nuclei with allowance for meson degrees of free-

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dom, the $\Delta(1236)$ resonance, three-particle forces, etc. (in this connection, we mention Refs. 8–13).

The relativistic theory of nuclear structure is being developed in two directions.

In the first, one investigates effective Lagrangians with a set of mesons whose parameters are fitted to reproduce the observed saturation properties of nuclear matter and finite nuclei, this being done either in the Dirac–Hartree approximation or in the Dirac–Hartree–Fock approximation. The most characteristic example is the Walecka model. This approach has achieved a high degree of sophistication and can describe quantitatively many properties of nuclear ground states.

The second approach is based on the Dirac–Brueckner–Hartree–Fock theory of nuclear structure. The point of departure in this case is provided by the vacuum NN forces (of the type of one-boson exchange potentials: OBEP), which are fitted to the description of NN scattering and the observed properties of the deuteron. The effective NN interaction in the medium—Brueckner's G matrix—is then completely determined and does not contain any more free parameters. In such an approach, the saturation properties are not parametrized but must be obtained directly on the basis of the vacuum potentials. A great achievement of this theory is that, in contrast to nonrelativistic Brueckner theory, it leads to a saturation point quite close to the empirical value (not lying on the so-called Coester line). Theories of this type have been developed for both infinite nuclear matter and finite nuclei.

Calculations have also been made to establish the extent to which relativistic models with parameters chosen to describe the ground states can describe satisfactorily the excited states of finite nuclei.

In this review, we consider the systematics of the properties of nuclear matter, finite nuclei, and also nuclear dynamics in the framework of relativistic theory.

1. WALECKA MODEL. GENERAL CHARACTERIZATION OF THE MODEL

The anomalously large spin–orbit coupling is a direct indication of the important role played by relativistic effects in nuclei. To explain this, we compare the nuclear and the atomic situations. In an atom, the spin–orbit potential is determined by the Thomas formula

$$U_{LS}=\frac{1}{4M^2}\frac{1}{r}\frac{dU}{dr}\mathbf{1}\cdot\boldsymbol{\sigma}, \tag{1}$$

where U is a central potential (Coulomb field of the nucleus plus self-consistent field), and M is the mass of the free nucleon.

Application of Eq. (1) to a nucleus leads, first, to an incorrect sign of the effect (the central potential of the nucleus—a well of depth 50 MeV and radius of the order of the nuclear dimension—has a positive derivative, whereas the observed sign is negative) and, second, to an absolute magnitude of the effect that is approximately 30 times smaller than is observed. Therefore, in the case of a nucleus it is necessary to introduce in Eq. (1) a negative coefficient that is numerically equal to 30 and characterizes the enhancement of the relativistic effects in nuclei.[14]

A solution to the problem was proposed by Duerr,[15] who described the nucleus by a Dirac equation of the form ($\hbar=c=1$)

$$[\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M+\beta S(r)+V(r)]\psi_\lambda=E_\lambda\psi_\lambda, \tag{2}$$

where the matrices α and β have the traditional representation, E_λ are the eigenvalues, and ψ_λ are the eigenfunctions:

$$\psi_\lambda=\begin{pmatrix}\varphi_\lambda\\ \chi_\lambda\end{pmatrix}. \tag{3}$$

As can be seen from Eq. (2), in Duerr's relativistic theory the nuclear nucleons move in two fields, one of which, $S(r)$, is a world scalar, while the other, $V(r)$, transforms as a relativistic vector [$V(r)$ is the time component of a 4-dimensional vector field]. It also follows from Eq. (2)[15] that the central potential in which the nucleons move is determined by the sum of the fields $S(r)$ and $V(r)$, whereas the spin–orbit potential corresponding to this equation is determined by the gradient of their difference: $V(r)-S(r)$. If we take the depth of the attractive field to be ~ -400 MeV and the magnitude of the repulsive vector field to be $\sim +300$ MeV, then we can recover to a good qualitative accuracy both the depth of the nuclear shell potential and the strength of the spin–orbit coupling in the nucleus.[15]

Duerr's paper was published in 1956, when apart from the pion no other mesons were known experimentally. For this reason, the paper appeared academic and was not developed at that time. However, in 1961–1962 vector mesons were discovered experimentally, and at the beginning of the seventies there appeared the models of one-boson exchange (OBEP),[3,8,9,11] in the framework of which it was shown to be necessary to introduce scalar mesons in order to describe the attraction between nucleons at intermediate distances. This has had the consequence that in recent years there has been a strong growth of interest in the relativistic treatment of the properties of nuclear matter and finite nuclei. The number of studies in this direction is increasing.

At the present time, Walecka's model[16] is widely used. The main reason for the success of this model is its simplicity. It is based on the Lagrangian

$$\begin{aligned}\mathfrak{L}=&\bar{\psi}(i\gamma^\mu\partial_\mu-M)\psi+\tfrac{1}{2}\partial_\mu\sigma\cdot\partial^\mu\sigma-U(\sigma)-\tfrac{1}{2}\partial_\mu V_\nu\partial^\mu V^\nu\\&+\tfrac{1}{2}m_V^2V_\mu V^\mu-g_S\bar{\psi}\psi\sigma-g_V\bar{\psi}\gamma^\mu\psi V_\mu\end{aligned} \tag{4}$$

with the potential function

$$U(\sigma)=\frac{1}{2}m_S^2\sigma^2+\frac{1}{2}\lambda m_S^2\frac{g_S}{M}\sigma^3+\frac{1}{8}\mu m_S^2\left(\frac{g_S}{M}\right)^2\sigma^4,$$

where σ and V_μ are scalar and vector meson fields ($\mu=0$, 1, 2, 3); ψ is the nucleon field; γ_λ are the traditional Dirac matrices;[17] g_S and g_V are the constants of the coupling of the scalar and vector mesons with the nucleons; and m_S, m_V, M are the masses of the scalar meson, the vector meson, and the nucleon [in Walecka's model, the isoscalar vector meson is identified with the ω meson ($J^P I = 1^-, 0$), and therefore in all the following calculations its mass is taken to be $m_\omega = 783$ MeV].

In the model described by the Lagrangian (4), the neutral vector meson interacts with the baryonic current

$$B_\lambda = \bar{\psi}\gamma_\lambda\psi,$$

and the scalar meson interacts with the scalar density $\bar{\psi}\psi$.

The Lagrangian (4) corresponds to Walecka's model for $\lambda = \mu = 0$, while for $\lambda = \mu = 1$ it reproduces the Lagrangian of the nonlinear σ model. The σ model itself does not give the correct saturation properties of nuclear matter. For this reason, λ and μ were regarded in Refs. 18–27 as parameters, and the functional $U(\sigma)$ was described in the form

$$U(\sigma) = \frac{1}{2} m_S^2 \sigma^2 + \frac{1}{3} b\sigma^3 + \frac{1}{4} c\sigma^4. \tag{5}$$

The description of the properties of nuclear structure on the basis of Walecka's model puts nuclear theory on a qualitatively new level, at which it can be regarded as a relativistic quantum field theory. Of course, the exact relativistic quantum-field description corresponding to Walecka's Lagrangian is very complicated. However, it turns out that in the framework of this model it is possible to obtain many important results by using the approximation of an average field, in accordance with which the fields that occur in Walecka's model are treated as classical fields. Generally speaking, this approximation requires a theoretical justification, which was provided by Walecka only in the limit of high densities. The use of the model in the mean-field approximation at the observed nuclear densities is at present justified only by the large number of experimental results that can be reproduced in the framework of Walecka's model in this approximation. In what follows, it will be this approximation that we shall use. In it, the model describes the following set of facts in the framework of a unified approach: a) the saturation property[16] (see also, for example, Refs. 28–35); b) in the high-density approximation, Zel'dovich's ultrarelativistic limit[36] for the equation of state; c) the spin–orbit interaction in nuclei (Refs. 15, 34, and 37–40); d) the energy dependence of the real part of the optical potential.[41–44]

It should be emphasized that even the reproduction of the saturation property, i.e., the fact that for infinite nuclear matter the correct binding energy per nucleon $E/A = -15.75$ MeV at the "empirically observed" density $n_0 = 0.19$ fm^{-3} is obtained, is an important result; the simultaneous description of a large body of experimental data is an undoubted success of Walecka's model.

We note now that if the scattering of two baryons in free space is calculated using the ladder approximation for the Bethe–Salpeter equation, then the interaction contained in Walecka's Lagrangian ($b = c = 0$) can be replaced by an equivalent potential in the momentum space:[16]

$$V(q)_{\text{eq}} = g_V^2 \frac{\gamma_\lambda^{(1)}\gamma_\lambda^{(2)}}{q^2 + m_V^2 - i\eta} - g_S^2 \frac{\mathbf{1}^{(1)}\mathbf{1}^{(2)}}{q^2 + m_S^2 - i\eta},$$

where $q_\lambda = (-\mathbf{q}, q_0)$ is the four-dimensional momentum transfer, and $\gamma_\lambda^{(1,2)}$ and $\mathbf{1}^{(1,2)}$ are matrices ($\mathbf{1}$ is the unit matrix) referring to the first and second particles. If the baryons are assumed to be heavy and move nonrelativistically, then we can use the approximations

$$\gamma_\lambda^{(1)}\gamma_\lambda^{(2)} \to \mathbf{1}^{(1)}\mathbf{1}^{(2)}, \quad |q_0| \ll |\mathbf{q}|,$$

which lead to a potential that does not take into account retardation, does not depend on the spins, and in the coordinate representation has the form

$$\frac{1}{4\pi}\left(g_V^2 \frac{e^{-m_V r}}{r} - g_S^2 \frac{e^{-m_S r}}{r}\right). \tag{6}$$

If $g_V^2 > g_S^2$, this potential is repulsive at short distances; if $m_V > m_S$, then the potential (6) is attractive at large distances. Thus, the interaction contained in (6) includes the main features of the nucleon–nucleon potentials that are responsible for saturation.

It is true that in (4) there is no one-pion tail. To some extent, its absence can be justified by the fact that in the case of 0^+ symmetry only scalar and vector fields can contribute to the ground state in the Hartree approximation.

For the Lagrangian (4), the Euler–Lagrange equations

$$\frac{\partial \mathfrak{L}}{\partial \varphi} - \frac{\partial}{\partial x^\mu}\frac{\partial \mathfrak{L}}{\partial \varphi_\mu} = 0;$$

$$\varphi_\mu = \frac{\partial \varphi}{\partial x^\mu}, \quad \mu = 0, 1, 2, 3, \tag{7}$$

have the form (for $b = c = 0$)

$$[\gamma^\mu(i\partial_\mu - g_V V_\mu) - M - g_S\sigma]\psi = 0, \tag{8}$$

$$(\Box - m_S^2)\sigma = g_S\bar{\psi}\psi, \tag{9}$$

$$(\Box - m_V^2)V^\nu = -g_V\bar{\psi}\gamma^\nu\psi. \tag{10}$$

In what follows, we shall be interested only in static solutions; in this case the wave functions of the nucleons ψ_λ (where λ is a complete set of quantum numbers) and their energies E_λ satisfy the Dirac equation introduced by Duerr:

$$E_\lambda\psi_\lambda = [\boldsymbol{\alpha}\cdot\mathbf{p} + V(r) + \beta(M + S(r))]\psi_\lambda, \tag{11}$$

while the vector, V, and scalar, S, fields satisfy Klein–Gordon–Fock equations [$S(r) = g_S\sigma(r)$, $V(r) = g_V V_0(r)$]:

$$(\Delta - m_V^2)V(r) = -g_V^2 n(r),$$

$$n(r) = \sum_\lambda n_\lambda \bar{\psi}_\lambda\beta\psi_\lambda = \sum_\lambda n_\lambda(|\varphi_\lambda|^2 + |\chi_\lambda|^2), \tag{12}$$

$$(\Delta - m_S^2)S(r) = g_S^2 n_S,$$

$$n_S(r) = \sum_\lambda n_\lambda\bar{\psi}_\lambda\psi_\lambda = \sum_\lambda n_\lambda(|\varphi_\lambda|^2 - |\chi_\lambda|^2), \tag{13}$$

where φ_λ and χ_λ, respectively, are the large and small components of the bispinor. It can be seen from Eqs. (12) and (13) that the vector field $V(r)$ is repulsive, and the scalar field $S(r)$ is attractive. We note that for infinite nuclear matter and in the ground states of even–even nuclei the spatial components V^i of the vector field are absent.

The properties of the ground state of relativistic nuclear matter were considered in Ref. 16. They can be obtained from the Lagrangian (4) (we shall first consider the Lagrangian without the self-interaction of the scalar field: $b=c=0$). By virtue of isotopic symmetry, only isoscalar fields are present in the ground state. The ground-state properties are determined by the dimensionless parameters $M^2g_i^2/m_i^2$. In accordance with Walecka's model,[16] these parameters are

$$C_S^2=M^2\frac{g_S^2}{m_S^2}=266.9;\quad C_V^2=M^2\frac{g_V^2}{m_V^2}=195.7. \tag{14}$$

For these values, the equilibrium density of the ground state is $n_0=0.19\ \mathrm{fm}^{-3}$, and the binding energy per nucleon is $E/A=-15.75$ MeV. The quasiparticle energy is[16,45]

$$E_k=g_VV_0+e_k,\quad e_k=\sqrt{k^2+\mathfrak{M}^2},\quad \mathfrak{M}=M+g_S\sigma, \tag{15}$$

where V_0 is the time component of the vector-meson field, and $\mathfrak{M}$ is the effective nucleon mass. The fields that form the ground state have the form

$$g_VV_\mu=\delta_{\mu0}\frac{g_V^2}{m_V^2}n=\delta_{\mu0}\frac{g_V^2}{m_V^2}\sum_{\lambda,|\mathbf{k}|<k_F}\bar{\psi}_{\mathbf{k},\lambda}\gamma^4\psi_{\mathbf{k},\lambda}, \tag{16}$$

$$g_S\sigma=-\frac{g_S^2}{m_S^2}n_S=-\frac{g_S^2}{m_S^2}\sum_{\lambda,|\mathbf{k}|<k_F}\bar{\psi}_{\mathbf{k},\lambda}\psi_{\mathbf{k},\lambda}, \tag{17}$$

where $\mathbf{k}$ and λ denote the momentum and polarization of the quasiparticle, and k_F is the Fermi momentum. The sums on the right-hand sides of Eqs. (16) and (17) include protons and neutrons. Such sums are calculated in accordance with the rule[46]

$$\sum_{\lambda,|\mathbf{k}|<k_F}\bar{\psi}_{\mathbf{k},\lambda}O\psi_{\mathbf{k},\lambda}=\int_{|\mathbf{k}|<k_F}\frac{d^3\mathbf{k}}{(2\pi)^3e_k}\operatorname{tr}\,O(\hat{k}+\mathfrak{M}), \tag{18}$$

where O is an arbitrary operator, and $\hat{k}=\gamma^\mu k_\mu$, $k_\mu(e_k,\mathbf{k})$. Setting $O=\gamma^4$, we obtain the traditional expressions for the density (time component of the nuclear current):

$$n=\int_{|\mathbf{k}|<k_F}\frac{d^3\mathbf{k}}{(2\pi)^3}\operatorname{tr}\frac{\gamma^4(\hat{k}+\mathfrak{M})}{e_k}=\frac{2k_F^3}{3\pi^2}. \tag{19}$$

Setting $O=1$, we obtain the following expression for the scalar density:

$$n_S=\int_{|\mathbf{k}|<k_F}\frac{d^3\mathbf{k}}{(2\pi)^3}\operatorname{tr}\frac{\hat{k}+\mathfrak{M}}{e_k}$$
$$=\frac{\mathfrak{M}}{\pi^2}\left(k_F\sqrt{k_F^2+\mathfrak{M}^2}-\mathfrak{M}^2\ln\frac{k_F+\sqrt{k_F^2+\mathfrak{M}^2}}{\mathfrak{M}}\right). \tag{20}$$

This is an equation for n_S, since in accordance with (15) and (17)

$$\mathfrak{M}=M-\frac{g_S^2}{m_S^2}n_S. \tag{21}$$

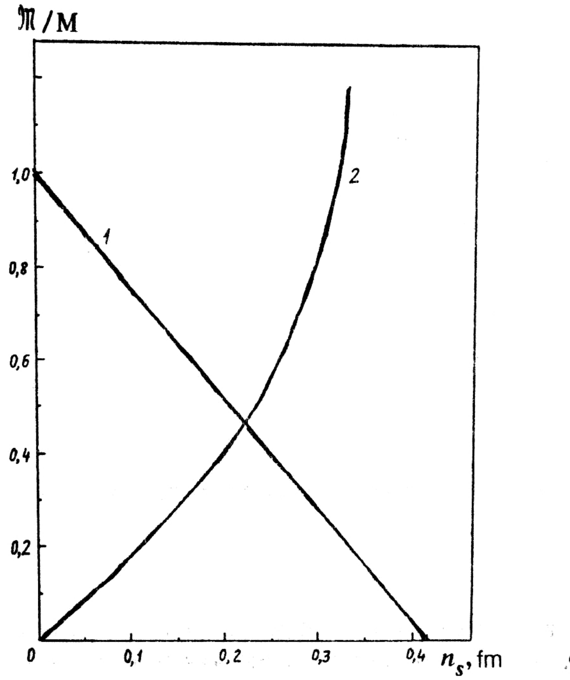

FIG. 1. Dependence of the ratio $\mathfrak{M}/M$ (curve 1) and the density n (curve 2) on the scalar density n_S.[45]

Solving Eq. (20), we obtain the dependence of n_S and $\mathfrak{M}/M$ on the density n, which is shown in Fig. 1. It is very important that the effective mass is a decreasing function of the density.[45] In the ultrarelativistic limit ($n\to\infty$), $\mathfrak{M}=0$. Indeed, as follows from (20), the ultrarelativistic scalar density is

$$n_{S\infty}=\frac{m_S^2}{g_S^2}M=0.4025\ \mathrm{fm}^{-3}. \tag{22}$$

For ground-state density $n_0=0.19\ \mathrm{fm}^{-3}$, we have

$$n_{S0}=0.176\ \mathrm{fm}^{-3},\quad \frac{\mathfrak{M}_0}{M}=0.56. \tag{23}$$

Thus, the "observed" scalar density is almost half the ultrarelativistic value. The significance of this fact is manifested[45] in the response of a system to weak external fields. In such fields, virtual particle–hole pairs are excited. In accordance with Eq. (15), the energy of such a pair ($|k_1-k_2|\ll k_F$) is

$$\begin{aligned}E_{\mathrm{ph}}&=E_{k_1>k_F}-E_{k_2<k_F}\\&=\sqrt{k_1^2+\mathfrak{M}^2}-\sqrt{k_2^2+\mathfrak{M}^2}\\&\approx\frac{k_F}{\sqrt{k_F^2+\mathfrak{M}^2}}(k_1-k_2)\\&=\left(\frac{v_0}{c}\right)_{\mathrm{eff}}(k_1-k_2).\end{aligned} \tag{24}$$

As can be seen from Eq. (24), the vector field does not occur in E_{ph}, and, thus, the behavior of the system in external fields is determined solely by the scalar field through

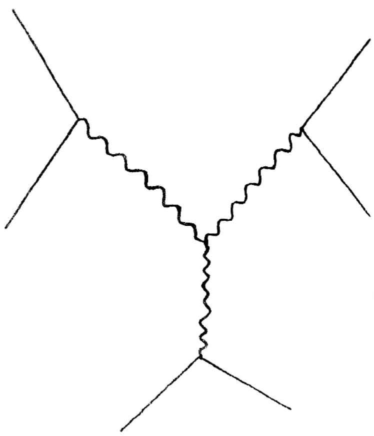

FIG. 2. Mechanism of three-particle forces due to exchange of a scalar meson (wavy line).

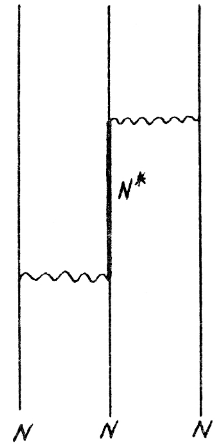

FIG. 3. Traditional three-particle forces in nuclei.

the effective nucleon mass. Setting $k_F=1.42\ \mathrm{fm}^{-1}$ and $\mathfrak{M}/M=0.56$ in (24), we obtain the following value of the nucleon velocity on the Fermi surface:

$$\left(\frac{v_0}{c}\right)_{\mathrm{eff}}=\frac{k_F}{\sqrt{k_F^2+\mathfrak{M}^2}}=0.47, \qquad (25)$$

which is not small compared with the speed of light c.

The terms $(1/3)b\sigma^3$ and $(1/4)c\sigma^4$ in (4) correspond to allowance for the self-interaction of the scalar field (see Refs. 19 and 47, and also Ref. 48, which is a study of the influence of the self-interaction of the scalar mesons that is inherent in the σ model on the equation of state of a high-density system of nucleons; in the σ model, the term with σ^3 corresponds to an attractive interaction, and the term with σ^4 to a repulsive interaction). We note that a relativistic approach of this kind with allowance for self-interaction is analogous to nonrelativistic calculations of finite nuclei and nuclear matter using density-dependent interactions.[49,50] In particular, it was shown in Ref. 50 that the two-particle density-dependent interaction

$$v_{12}=\frac{1}{6}t_3(1+P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2)n\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right) \qquad (26)$$

and the effective zero-range three-particle forces

$$v_{123}=t_3\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_2-\mathbf{r}_3) \qquad (27)$$

are equivalent in Hartree–Fock calculations. The contribution of the three-particle forces (27) to the binding energy per nucleon for nuclear matter is (Ref. 56) $(1/16)t_3n^2$, i.e., 15–20 MeV per nucleon.

On the other hand, as is shown in Ref. 51,[1)] a self-interaction proportional to the third power of the scalar field also leads to three-particle forces (Fig. 2) (the more traditional mechanism of three-particle forces is shown in Fig. 3; Ref. 51); its contribution to the expression for the energy density is determined by the functional $(b/3V)\int\sigma^3d^3r$ (V is the volume of the system), which in Ref. 5 was estimated by perturbation theory using as the field $\sigma(r)$ the solution of the problem in which only the quadratic term is contained in the potential function $U(\sigma)$ [see (4)].

Using also a short-range approximation, Barshay and Brown[51] estimated (without allowance for the quantum corrections) the contribution of the three-particle forces (Fig. 2) to the binding energy per nucleon for the normal phase; the estimates were made using the parameters of the σ model $[(1/3)b=m_S^2/2\,f_\pi,\ f_\pi=95\ \mathrm{MeV}]$ and give a value of approximately 15 MeV for the energy of attraction. This value is too large. Such large three-particle forces must significantly influence the properties of the triton, but it is well known that the triton energy can be reproduced rather well by means of two-particle forces obtained by fitting nucleon–nucleon scattering and the deuteron properties. In Ref. 52, some approximations in the calculations of Ref. 51 are noted, and it is pointed out that their elimination can lead to a significant reduction in the contribution of the three-particle forces. Thus, Barshay and Brown did take into account partly the correlations[51] but ignored the exchange terms altogether, whereas it is known[53] that exchange terms alone in the absence of correlations at low densities reduce the three-particle forces by a factor of 3/8.

It was shown in Ref. 54 that allowance for the quantum corrections has the consequence that the three-particle forces (Fig. 2) act only in the anomalous phase (in this case, an anomalous state of Lee–Wick type[10,19,55,56] occurs at a density that is about an order of magnitude greater than the normal density, but this state is not bound—the energy of the "excitation" per nucleon is about 100 MeV). Thus, at high densities a bound anomalous state can occur only if large three-particle forces act in the normal phase.

Inclusion of the σ^4 self-interaction gives an additional repulsion. This leads to a decrease of the constant of the coupling between the vector ω meson and the nucleon and to better agreement between the compression modulus K calculated in such a model and its experimental value.

In Ref. 19, Boguta and Bodmer made relativistic calculations for infinite and semi-infinite symmetric nuclear matter in the framework of the Hartree and Thomas–Fermi approximations (in the case of infinite nuclear matter, these two approximations are the same). The parameters of the model are the coefficients of σ^3 and σ^4, the constants of the coupling of the scalar and the vector mesons, and the mass of the scalar meson. These parameters are chosen to describe the "empirical" properties of nuclear matter (binding energy per nucleon, density at saturation) and the nuclear surface. The investigation in Ref. 18 was made in the approximation of a "mean field." In Ref. 2, the limits of applicability of this approximation are discussed, and the role of correlations in Walecka's model is considered.

We mention also the many-body theory based on Walecka's Lagrangian constructed in Ref. 57. This theory is used to describe a high-density system of interacting baryons at $T=0$. In Refs. 18 and 19, the relativistic field-theoretical formalism of Green's functions at $T=0$ is used.

In order to determine whether or not the considered model many-baryon system remains stable in the high-density limit, a study was made in Ref. 57 of its collective modes corresponding to microscopic vibrations of the density. In the high-density approximation, these modes are determined by the vector interaction, and the spectra of the collective modes corresponding to the density fluctuations are given by the poles of the propagator of the vector meson. The relativistic plasma frequencies Ω are given by the expression $\Omega^2=g_V^2k_F^3/3\pi^2\mathscr{E}_F$, where $\mathscr{E}_F=(k_F^2+M^2)^{1/2}$. The longitudinal mode consists of two branches, namely, a high-frequency meson branch, which can be likened to a traditional longitudinal plasma collective mode but with frequency $q_0=(m_V^2+\Omega^2)^{1/2}$, and a low-frequency acoustic branch, which can be identified with zero sound.

The formalism of relativistic finite-temperature Green's functions was used in Ref. 58 (and also in the more detailed study of Ref. 59) to calculate the equation of state of baryonic matter interacting with scalar and vector fields[2)] (Walecka's model). In Ref. 58, an equation of state for any temperature and density of the baryons was obtained; with increasing baryon density, this equation tends asymptotically to the maximally hard equation $P=\varepsilon$ (ε is the energy density and P is the pressure); the quantum corrections make the equation of state softer, i.e., they make the ratio P/ε smaller. It is noted in Ref. 58 that a phase transition of the gas–liquid type takes place in the system at low temperatures, the transition disappearing at temperatures above a critical value $T_c \gtrsim 10^{12}$ K.

In Ref. 61, the formalism of finite-temperature Green's functions was used to obtain an expression for the coefficient of thermal conductivity κ of nuclear matter with allowance for relativistic effects, which change the dependence of κ on the density n of the nuclear matter: In the nonrelativistic approximation $\kappa\sim n^{5/3}$, while in the relativistic approximation (at not too small n) $\kappa\sim n$.

The answer to the question of the manner in which the mesons interact with the nucleons in the nuclear medium depends strongly on the density of the medium—at certain critical densities, phase transitions can take place in the medium (Refs. 47, 55, 56, and 62).

2. RELATIVISTIC DESCRIPTION OF THE PROPERTIES OF NUCLEAR MATTER. DETAILS OF CALCULATIONS

We have considered above general properties of Walecka's model. In this section, we shall dwell on some details of the relativistic calculations of the properties of nuclear matter in the framework of Walecka's model or modifications of it. In what follows, we shall consider the results obtained in the framework of an analogous scheme for finite nuclei. In this section, we shall use the mean-field approximation, taking the Lagrangian (4) as our point of departure.

We shall consider the results obtained in Ref. 27. An advantage of this study was that its authors succeeded for the first time in relativistic calculations in obtaining the correct value of the compression modulus besides other properties of nuclear matter as well as giving a good description of the ground states of spherical nuclei.

In the case of isoscalar nuclear matter, we obtain, using the Lagrangian (4) and the functional $U(\sigma)$, the following equations for the vector and scalar fields:

$$-\frac{\partial U}{\partial\sigma}=g_S n_S, \tag{28a}$$

$$m_V^2 V_0=g_V n; \tag{28b}$$

at the same time, the nucleon field satisfies the Dirac equation (11), the effective mass of the nucleon is determined by the expression (15), and the densities n_S and n are determined by Eqs. (12) and (13).

The equation of state of nuclear matter is given by a parametric form, the energy density being determined by the expression

$$\varepsilon=\frac{1}{2}\frac{C_V^2}{M^2}n^2+U(\sigma)+\frac{4}{(2\pi)^3}\int_0^{k_F}d^3k(k^2+\mathfrak{M}^2)^{1/2}, \tag{29}$$

and the pressure $P=n^2(\partial\varepsilon/n)/\partial n$ by

$$P=\frac{1}{2}\frac{C_V^2}{M^2}n^2-U(\sigma)+\frac{1}{3}\frac{4}{(2\pi)^3}\int_0^{k_F}d^3k(k^2+\mathfrak{M}^2)^{-1/2}. \tag{30}$$

From the equation of state, we can readily obtain an analytic expression for the compression modulus:

$$K=9n^2\frac{\partial^2\varepsilon/n}{\partial^2 n}\Big|_{n=n_0}, \tag{31}$$

where n_0 is the density corresponding to saturation. For the bulk modulus, we also have

$$\frac{1}{9}K=\frac{k_F^{(0)2}}{3\nu}+\left(\frac{C_V^2}{M^2}\right)n_0+\frac{\mathfrak{M}n_0}{\nu}\frac{\partial\mathfrak{M}}{\partial n}\Big|_{n=n_0}. \tag{32}$$

From the self-consistency equations (28a) and (15) we obtain

$$\frac{\partial\mathfrak{M}}{\partial n}=\frac{\mathfrak{M}}{\nu}\left(\frac{3n}{\nu}+\frac{3}{g_S}\frac{1}{\mathfrak{M}}\frac{\partial U}{\partial\sigma}-\frac{1}{g_S^2}\frac{\partial^2 U}{\partial\sigma^2}\right)^{-1} \tag{33}$$

for

$\nu=(k_F^2+\mathfrak{M}^2)^{1/2}$.

It should be emphasized that all the quantities in Eq. (32) are calculated at the saturation point.

The equation of state, Eqs. (29) and (30), the compression modulus, and Eqs. (32) and (33) depend, as in the case of the Walecka model itself,[16] on the dimensionless parameters $C_i^2=(g_i/m_i)^2M^2$ ($i=S, V$), but also on the coefficients of σ^3 and σ^4 in the functional $U(\sigma)$. It is also convenient to introduce the two dimensionless parameters[24,26,27]

$$\bar{b}=b/Mg_S^3,\quad \bar{c}=c/g_S^4, \tag{34}$$

which are related to the coupling constants of the other calculations[21,23] by the equations

$$\bar{b}=3\lambda/2C_S^2,\quad \bar{c}=\mu/2C_S^2, \tag{35}$$

$$\bar{b}=-b_{\mathrm{BGG}},\quad \bar{c}=c_{\mathrm{BGG}}, \tag{36}$$

where the subscript BGG corresponds to Refs. 21 and 23.

At the same time, it is possible to obtain the key equation

$$M^2(\mathfrak{M}-M)/C_S^2+\bar{b}M(\mathfrak{M}-M)^2+\bar{c}(\mathfrak{M}-M)^3=-n_S, \tag{37}$$

which relates all the dimensionless parameters of the considered problem.

Thus, the nonlinear σ model, augmented by the vector meson, contains four dimensionless parameters C_S^2, C_V^2, $\bar{b}$, $\bar{c}$ and three quantities—the energy density (29), the pressure (30), and the compression modulus (32)–(37), each of which, obviously, depends on the effective mass $\mathfrak{M}$, determined by Eq. (37).

In order to reproduce the saturation property of nuclear matter,

$$\left.\begin{array}{r}\varepsilon/n-M=-a_1\\ P=0\end{array}\right\}\quad \text{for}\quad k_F=k_F^{(0)}\ (\text{or } n=n_0), \tag{38}$$

where a_1 and n_0 are the empirical values of the binding energy and the density, and also the compression modulus obtained from data on giant monopole resonances,

$$K=K_\infty, \tag{39}$$

certain restrictions must be imposed on the existing parameters. However, it is obvious that additional information is needed for the unique determination of the four dimensionless parameters. We shall discuss this question below.[27]

Symmetry energy

The first possibility is associated with calculating the coefficient of the symmetry energy. To this end, we must, in addition to the isoscalar, scalar, and vector fields, also introduce an isovector vector meson field b_μ. For this, we must add to the Lagrangian (4)

$$\mathfrak{L}_1=\tfrac{1}{2}m_b^2b_\mu b^\mu-\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}-\tfrac{1}{2}g_b\bar{\psi}\gamma_\mu\tau b^\mu\psi \tag{40}$$

with

$$B_{\mu\nu}=\partial_\mu b_\nu-\partial_\nu b_\mu.$$

Actually, in the mean-field approximation for nonsymmetric nuclear matter the entire contribution of the b-meson field is associated exclusively with its time component ($\mu=0$), and in the equation of state both the energy density and the pressure will include the term

$$\frac{C_b^2n_3^2}{2M^2}, \tag{41}$$

where $C_b=g_b(m_b/M)^{-1}$, m_b is the mass of the isovector vector meson, and $n_3=n_p-n_n$ is the difference between the proton and neutron densities. Then the coefficient of the symmetry energy is readily calculated[63,64] and can be expressed as the sum of a "kinetic part" and an isovector contribution:

$$a_4=(a_4^{(1)}+a_4^{(2)})=\frac{1}{6}\frac{k_F^{(2)}}{\nu}+\frac{1}{8}\frac{C_b^2}{M^2}n. \tag{42}$$

This quantity must be fitted to the corresponding coefficient of the semiempirical mass formula.

However, one of the defects of such an approach is that a new dimensionless parameter appears in nuclear matter: C_b. Thus, the additional restriction associated with the need to reproduce the symmetry energy led to the appearance of a new parameter. Usually, the problem is solved by identifying the isovector meson b with the ρ meson, and its coupling constant is determined from the decay $\rho\to 2\pi$. However, this procedure was not regarded as very well founded in Ref. 27. The fact is that some authors treat the coupling constants C_S and C_V of the scalar and vector mesons as effective, using them as free parameters. On the other hand, the ρ meson is introduced into this scheme with the vacuum values of the parameters. However, it was established that on the transition from nuclear matter to finite nuclei the vacuum value of the ρ-meson coupling constant is too small. An additional fitting of a constant is needed to obtain the correct value of the isobar-spin potential.[65] In this case, it is better to treat C_b as a free parameter like C_S and C_V and to fit a_4 to the empirical value, varying C_b as necessary. Such a point of view was adopted in Refs. 21, 27, 66, and 67. It gives a certain freedom in the choice of the coupling constants, since there are five dimensionless parameters and four restrictions. This point is important in reproducing the isobar-spin potential for nuclei with $N\neq Z$.[27]

Surface energy

The last important quantity for nuclear matter that we must consider[27] is the surface energy. This is of special interest for the transition to the problem of finite nuclei. In principle, to solve this problem one should start with the volume density of the total energy and calculate the surface tension. Then the coefficient of the surface energy can be written in the form

$$a_2=4\pi r_0^2\int_{-\infty}^{\infty}\left[\varepsilon(n)-\left(\frac{\varepsilon(n)}{n}\right)_{\text{n.m.}}n(z)\right]dz, \tag{43}$$

TABLE I. Properties of symmetric nuclear matter and dimensionless coupling constants in the linear model for different choices of the density at saturation (binding energy at saturation $E/A=15.75$ MeV).

Reference	k_F, fm^{-1}	n_0, fm^{-3}	C_S^2	C_V^2	$\mathfrak{M}/M$	K, MeV
[16]	1,42	0,1934	266,90	195,70	0,556	544
[27]	1,34	0,1625	323,64	244,53	0,546	546
[67]	1,325	0,1571	334,75	254,08	0,545	545
[66]	1,30	0,1484	357,47	273,87	0,541	541
[25]	1,29	0,1450	366,99	282,15	0,539	539

where r_0 is related to the density n_0 corresponding to saturation in nuclear matter, and the density $n(z)$ varies only along the z axis. The calculation can be made in the approximation of a local density, as in Ref. 19, with allowance for the contribution of the nonlinear σ terms. The complexity of the calculation in this case is practically the same as in the case of finite nuclei. At the same time, $\varepsilon(n)$ can be represented as the sum of a volume part and a gradient term, which arises from the expansion of the kinetic energy. The density profile is found by minimizing the surface tension.

To avoid a nonrelativistic reduction of the energy density (which may lead to ghost errors), the surface energy was determined as follows in Ref. 27. If a restriction is made for the case of nuclei with $N=Z$ without allowance for the Coulomb interaction, then for the total energy per particle for a given nucleus we have the expression

$$E/A=-a_1+a_2A^{-1/3}+a_3A^{-2/3}, \tag{44}$$

where a_3 is the coefficient associated with the curvature of the surface. In principle, calculations must be made for very large hypothetical nuclei in order to eliminate shell effects and effects due to curvature; in this case, the coefficient a_2 has the form

$$a_2=\lim_{A\to\infty}(E/A+a_1)A^{1/3}. \tag{45}$$

However, since finite $N=Z$ systems were considered in Ref. 27, approximate values of the coefficients a_2 and a_4 were obtained, and the shell effects were taken into account in accordance with Ref. 68.

In the consideration of finite systems, the calculated quantities will depend not only on the dimensionless constants C_S, C_V, C_b, $\bar{b}$, $\bar{c}$ but also on the meson masses. This leads to the appearance of two new parameters: m_S and m_V (and also m_b, the mass of the isovector vector meson, in the case of $N\neq Z$ nuclei). As a rule, the vector meson is identified with the ω meson, and the isovector vector meson with the ρ meson. Since the scalar meson is identified with the hypothetical σ meson, its mass remains an adjustable free parameter.

In Ref. 27, the following equilibrium properties of cold nuclear matter were chosen for fitting the parameters:

$$a_1=+15.75 \text{ MeV},$$

$$k_F^{(0)}=1.34 \text{ fm}^{-1} \quad (n_0=0.1625 \text{ fm}^{-3}). \tag{46}$$

The compression modulus was taken to be[69]

$$K_\infty=(210\pm30) \text{ MeV}. \tag{47}$$

The symmetry energy and surface energy per particle were chosen in accordance with the semiempirical estimates

$$a_4=33 \text{ MeV}, \quad a_2=18 \text{ MeV}. \tag{48}$$

The values of the masses are taken equal to the experimental values $M=938.9$ MeV (the mean value of the neutron and proton masses), $m_V=783$ MeV (ω meson), and $m_b=770$ MeV (ρ meson). Thus, in the study of Ref. 27 there are five dimensionless coupling constants, C_S, C_V, $\bar{b}$, $\bar{c}$, which are not related by any restrictions, and the constant C_b, which is chosen in order to obtain the correct value of the symmetry energy. As regards the mass of the scalar meson, we note that this mass determines the range of the interaction and also influences the nature of the rate of decrease of the nuclear characteristics in the surface region.[65,66,70]

In the study of the properties of nuclear matter, this parameter will be important only in calculation of the surface energy. However, it will have a radical significance in calculations of finite nuclei.

Before we turn to the consideration of the results in the nonlinear σ model, we note[27] that a comparison of the different calculations is somewhat difficult because whereas all authors take the binding energy in nuclear matter to be ~ 16 MeV, the Fermi energy corresponding to saturation is, unfortunately, chosen differently in different calculations. For this reason, to have an approximate picture of the scale of variations on the transition from one calculation to another, Table I gives for the case of the linear σ model ($\bar{b}=\bar{c}=0$) the values of the effective nucleon mass and the compression modulus for different choices of the saturation density (Refs. 16, 25, 27, 66, and 67). It can be seen from the table that whereas the dimensionless coupling constants C_S and C_V differ rather strongly, neither $\mathfrak{M}/M$ nor K differ by more than 3%. Table I can be helpful in the comparison of the results obtained by different authors.

We note the following important circumstance. For any value of the compression modulus K, the coefficient $\bar{c}$ can be either positive or negative. This means that if we require rigorously that the coefficient of σ^4 be positive (without contemplating a possible renormalization by vacuum fluctuations[3)]), then it is necessary to take a small value of C_V and, therefore, a large effective mass in order to obtain a sensible value of the compression model. This can be stated in the equivalent form: If it is required to obtain a smaller value of the effective mass ($\mathfrak{M}/M=0.7$–0.8), then a positive value of $\bar{c}$ leads to a compression modulus $K=280$–300 MeV. Just such a choice was made in Refs. 21 and 23 in the investigation of pion condensation. However, if it is assumed that the sign of $\bar{c}$ is free (as was done in Ref. 27, in which densities in the neighborhood of the saturation density n_0 were considered), then the best fitting associated with obtaining small values of K comparable with K_∞ (47) is achieved for a negative value of the coefficient $\bar{c}$. If the effective mass is specified in the interval $0.55\leqslant\mathfrak{M}/M\leqslant0.70$, then small values can be obtained for $\bar{c}=-3\cdot10^{-3}$.

It is also important to note that for a reasonable value of K the coefficient $\bar{b}$ is always negative, i.e., leads to a repulsive interaction $\sim\sigma^3$. The calculation made in Ref. 27 shows that $\bar{c}$ becomes negative when K is less than ~300 MeV (if one does not permit too large values of the effective mass). Thus, in the region of physical interest the coefficients $\bar{b}$ and $\bar{c}$ have the same order of magnitude, namely, in the range from $-3\cdot10^{-3}$ to $-5\cdot10^{-3}$.

In Ref. 27, the best fit was obtained with a negative value of the coupling $\sim\sigma^4$, and this does not correspond to general theoretical ideas (see the previous section, and also reviews of the properties of chiral models, in particular Refs. 51, 55, and 56). However, it is a fact that single-loop meson fluctuations[71] were not taken into account in Ref. 27. In Ref. 71, it is assumed that allowance for such fluctuations leads to a positive value of the coefficient $\bar{c}$. By virtue of this, the calculations made in Ref. 27 should be regarded as phenomenological and as having value at densities corresponding to saturation.

We also make some comments concerning the coefficient a_4 of the symmetry energy at saturation. Its first part $a_4^{(1)}$, corresponding to the contribution of the kinetic energy, depends only on the effective nucleon mass and is equal to 20 MeV for $\mathfrak{M}/M=0.6$. The second part $a_4^{(2)}$ corresponds to the contribution of the b meson, and at saturation it depends only on the coupling constant g_b. Therefore, if the effective mass is chosen, then the constant C_b is fitted by reproducing the empirical value of the coefficient a_4 [Eq. (48)]. In the region $0.55\leqslant\mathfrak{M}/M\leqslant0.70$ the result obtained in Ref. 27 was $75\leqslant C_b^2\leqslant100$, i.e., a value larger than is obtained on the basis of the $\rho\rightarrow2\pi$ decay, which gives $C_b^2=54.71$ (Ref. 63). This range of values is comparable with the one that is used to calculate finite nuclei in Ref. 66.

In conclusion, we discuss the fitting of the coefficient of the surface energy. This is very important in reproducing the properties of finite nuclei. As we have already noted, the coefficient a_2 (and a_3) depends strongly on the dimensionless coupling constants, and also on the mass m_S of the scalar meson. The calculation of Ref. 27 shows that a small $\mathfrak{M}/M$ is needed in order to reproduce the empirical value given by Eq. (48).

Calculations for nuclear matter show that it is better to take small values of the effective nucleon mass, the meson mass, and the compression modulus.[27] (This corresponds to a general result, in accordance with which the surface energy is approximately proportional to $\sqrt{K}$, and it is also in agreement with the result[66,67] of a linear model, in which the calculated large values of K are associated with a too large value of a_2.)

To conclude this section, we note that after the publication of the papers of Lee and his collaborators[55,56,72] there was a considerable growth of interest in the investigation of chiral models in the description of the structure of nuclear matter and finite nuclei (Refs. 51, 54, and 73–80).

From the theoretical point of view, chiral symmetry ensures that there are important restrictions on strong-interaction processes not only in empty space but also in a nuclear medium. In particular, in Refs. 54, and 74–80 it is emphasized that chiral models can include a new treatment of the saturation mechanism of nuclear matter. These studies consider the consequences of invariance of the Lagrangian describing interacting meson and baryon fields with respect to the chiral transformation in addition to the usual isotopic invariance. There exist different realizations of chiral symmetry (see, for example, Refs. 72, 77, and 81). In Refs. 79 and 82–84, the authors make a restriction to the realization of chiral symmetry in various versions of the σ–ω model. This makes it possible to establish the correspondence between the chiral approach and the Dirac phenomenology for nuclei. In Ref. 82, a chiral model that ensures saturation of nuclear matter was proposed (see also Refs. 83 and 84).

3. RELATIVISTIC SELF-CONSISTENT SHELL MODEL

We consider the problem of reproducing the properties of the ground states of nuclei in the framework of a relativistic Hartree approximation. Such a problem was solved in the studies of Refs. 20, 27, 28, 35, 37, 66, 67, 85, and 86 (see also the reviews of Refs. 1 and 6). In this review, we shall consider only spherical nuclei. The main features of the description of deformed nuclei in a relativistic theory can be found in the review of Ref. 7.

In the studies of Refs. 28, 35, and 37, the point of departure was potentials of one-boson exchange type, while in the studies of Refs. 14, 20, 27, 66, 67, 85, and 86 the authors used Walecka's model or various modifications of it, in particular the one proposed in Ref. 19.

It should be emphasized that every relativistic self-consistent calculation of finite nuclei must be accompanied by a preliminary fitting of the parameters of the model for nuclear matter. This is one of the reasons for the detailed treatment of the properties of nuclear matter in the relativistic theory in the previous section. Some of the authors in their calculations of the nuclear ground states did not take as their point of departure the saturation properties of infinite nuclear matter. This is the reason why the coupling constants that they used led to incorrect properties of nuclear matter and incorrect density distributions in finite nuclei.

The studies of Refs. 27, 66, 67, 85, and 86 are free of this shortcoming. The results of Refs. 28 and 37 were discussed in detail in the reviews of Refs. 3 and 11. In this review, we shall consider in more detail the results of the more recent studies of Refs. 27, 66, 67, 85, and 86.

Augmenting the Lagrangian (4) with the isovector vector meson [see (40)], and also taking into account the interaction of the protons with the electromagnetic field for the ground state of spherically symmetric even–even nuclei, we obtain the following system of coupled Euler–Lagrange equations:[27]

$$\{-i\boldsymbol{\alpha}\cdot\nabla+\gamma_0[M+g_S\sigma(r)]+g_V V_0(r)$$
$$+\tfrac{1}{2}g_B\tau_3 b_0(r)+\tfrac{1}{2}e(1+\tau_3)A_0(r)\}\psi=E\psi, \tag{49}$$

where $\tau_3=+1$ for protons, $b_0(r)$ is the time component of the isovector vector field, and $A_0(r)$ corresponds to the electromagnetic field, and

$$\frac{d^2\sigma(r)}{dr^2}+\frac{2}{2}\frac{d\sigma(r)}{dr}-\frac{\partial U(r)}{\partial\sigma}=g_S n_S(r), \tag{50a}$$

$$\frac{d^2V_0(r)}{dr^2}+\frac{2}{r}\frac{dV_0(r)}{dr}-m_V^2V_0(r)=-g_V n(r), \tag{50b}$$

$$\frac{d^2b_0(r)}{dr^2}+\frac{2}{r}\frac{db_0(r)}{dr}-m_b^2b_0(r)=-\frac{1}{2}g_b n_3(r), \tag{50c}$$

$$\frac{d^2A_0(r)}{dr^2}+\frac{2}{r}\frac{dA_0(r)}{dr}=-en_p(r). \tag{50d}$$

In Eqs. (50), the sources are time-independent spherically symmetric densities:

$$n_{p,n}(r)=\sum_{\substack{\alpha,n,p\\ \text{occup}}}\frac{2j_\alpha+1}{4\pi}\frac{G_\alpha^2+F_\alpha^2}{r^2}, \tag{51a}$$

$$n_3(r)=n_p(r)-n_n(r), \tag{51b}$$

$$n(r)=n_p(r)+n_n(r), \tag{51c}$$

$$n_S(r)=\sum_{\substack{\alpha,n,p\\ \text{occup}}}\frac{2j_\alpha+1}{4\pi}\frac{G_\alpha^2-F_\alpha^2}{r^2}, \tag{51d}$$

where the large, G_α, and small, F_α, components of the wave function are determined by the bispinor

$$\psi_\alpha=\frac{1}{r}\begin{pmatrix}G_\alpha\\ iF_\alpha\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\end{pmatrix}\Omega_\alpha, \tag{52}$$

where Ω_α ($\alpha=n,\ l,\ j,\ m$) are the spin and angular degrees of freedom. They satisfy self-consistent coupled Dirac–Hartree equations (we omit the index α for simplicity):

$$\frac{dG}{dr}+\kappa\frac{G}{r}=(E+M+S-V)F, \tag{53a}$$

$$\frac{dF}{dr}-\kappa\frac{F}{r}=(M-E+S+V)G, \tag{53b}$$

with scalar potential

$$S=g_S\sigma(r), \tag{54}$$

whereas the vector potential contains contributions of different components:

$$V=g_VV_0(r)+\tfrac{1}{2}\tau_3g_bb_0(r)+\tfrac{1}{2}(1+\tau_3)eA_0(r). \tag{55}$$

We can then write, using obvious notation,

$$V=\begin{cases}U_V+U_R+U_A & \text{for protons,} \quad (56a)\\ U_V-U_R & \text{for neutrons.} \quad (56b)\end{cases}$$

In Eq. (53),

$$\kappa=-(1+\mathbf{1}\cdot\boldsymbol{\sigma})=\bar{\omega}(j+\tfrac{1}{2}), \tag{57}$$

where $\bar{\omega}$ is determined from the relation $j=l-\frac{1}{2}\bar{\omega}$.

In Eq. (49), $E=M+\varepsilon$, where ε is the separation energy. In the mean-field approximation, the total nuclear energy of the Lagrangian (4), (5), and (40) is given by[66]

$$E_{\rm tot}=\sum_{\substack{\alpha,n,p\\ \text{occup}}}\varepsilon(2j_\alpha+1)-\frac{1}{2}\int dr\bigg\{g_Sn_S(r)\sigma(r)-2U(\sigma)$$
$$+\sigma(r)\frac{\partial U}{\partial\sigma}+g_VV_0(r)n(r)+\frac{1}{2}g_Bb_0(r)n_3(r)$$
$$+eA_0(r)n_p(r)\bigg\}. \tag{58}$$

The upshot is that, to describe the ground-state properties of the even–even spherical nuclei, we have a system of coupled nonlinear differential equations, which must be solved iteratively. Before we give the results of self-consistent calculations, we note some general properties of them.[27] The mean-square nuclear radius does not depend strongly on K. However, this is not so in the case of the binding energies. This is reflected in the fact that on the transition from $K=210$ to $K=350$ MeV the surface energy a_2 increases and, therefore, the binding energy decreases. The spin–orbit splitting increases with increasing m_S but is practically unchanged by variations of K.

The calculations made in Ref. 27 reveal the following:

a) To obtain reasonable values of the mean-square radii, the mass m_S of the scalar meson in the present theory must be taken to be ~ 500 MeV. Since the nuclear radii are intimately related to the saturation density in nuclear matter, the assertion just made is in agreement with the choice in Refs. 66 and 67, in which m_S is taken equal to 525 and 500 MeV for $n_0=0.1484$ and $0.1571\ \text{fm}^{-3}$, respectively.

b) To obtain a value of the compression modulus close to the experimental value, a small nucleon effective mass $\mathfrak{M}$ in nuclear matter is needed. Otherwise, the calculations lead to binding energies per particle that are too low. For example, for $K=210$ MeV and $\mathfrak{M}/M=0.55$ the binding energy is approximately the same as for $K=280$ MeV and $\mathfrak{M}/M=0.65$. The main difference between these two cases confirms the spin–orbit splitting. (We are referring here to the ^{16}O nucleus and the $1p_{1/2}$–$1p_{3/2}$ spin–orbit splitting.)

TABLE II. Parameters of the nonlinear model (two possible sets are presented). The masses of the vector mesons are fixed: $m_V=783$ MeV (ω meson), $m_b=770$ MeV (ρ meson). The quantities in the brackets correspond to the coupling constants $g_i^2/4\pi$ for $i=S,\ V,\ b$ (Ref. 27).

Set	C_S^2	C_V^2	C_b^2	$\bar{b}$	$\bar{c}$	m_S, MeV
I	305,53 (6,09)	200,00 (11,07)	83,56 (4,47)	−0,002353	−0,002857	470
II	353,89 (7,51)	238,58 (13,20)	73,94 (3,96)	−0,002165	−0,003142	485

In summary, reasonable properties of the nuclear ground states are to be expected when the mass of the scalar meson is chosen in the neighborhood of $m_S=480$ MeV.

c) In Ref. 27, the problem of finding an optimal parameter set was not posed. Various possible sets that lead to approximately the same properties on the transition from nuclear matter to finite nuclei were found. Table II gives two possible sets of dimensionless coupling constants chosen in Ref. 27 to represent the results of the calculations.

Table III gives the nuclear-matter parameters discussed in the previous sections for the two sets of parameters.

The interest in set I is related to the value of the effective mass, but this set leads to excessively small spin–orbit splittings of the levels, as was noted above. Set II gives a better value of the compression modulus but an excessively small value of $\mathfrak{M}$ compared with the effective nucleon mass in the standard model of Walecka.

Table IV shows[27] the single-particle spectra for the ^{16}O, ^{40}Ca, and ^{48}Ca nuclei. The spectra are compared with the experimental values. The level positions are predicted fairly well by the model, but in these calculations (with the parameter set I) the spin–orbit splittings are found to be approximately 20% smaller than the experimental splittings. This is due to the use of set I, for which the effective nucleon mass is $\mathfrak{M}=0.62M$. The calculations made with set II give better agreement with experiment (see Ref. 27). It is obvious that the spin–orbit splittings are described better in the theory with the smaller value of the effective mass.

In Ref. 27, particular attention was devoted to comparison with experiment of a global property such as the total binding energy per particle. The reason for this is that in Ref. 27 the coefficient a_2 of the surface energy is reproduced rather well. In the comparison of the nuclear binding energies with experiment, the corrections for the center-of-mass motion from Ref. 87 were taken into account. The results of the calculations and the experimental values of the binding energies per particle for different nuclei are presented in Table V. The agreement between the theory and experiment is very good, but it is clearly better for set I than for set II. However, it is also obviously necessary to compare the mean-square radii before making a final assertion that there is good agreement between the theory and experiment.

The final value, including the correction for the center-of-mass motion, for each of the parameter sets must be compared with the experimental value.

Table VI gives the neutron and proton radii, or, rather, $\langle r_n^2\rangle^{1/2}-\langle r_p^2\rangle^{1/2}$, and also the charge radii.

It can be seen from Table VI that the mean-square radii are reproduced very well. In Ref. 27, the charge density distributions are given for the same nuclei. There is good agreement with experiment. In particular, the decrease of the density at the nuclear surface is well reproduced, but, of course, this is not surprising, since the surface energy is reproduced rather well in Ref. 27. The calculations in Ref. 27 were made for two parameter sets. It is clear that set I reproduces the nuclear surface better. This means that in the considered theory it is difficult to reproduce well the distribution of the charge density for a compression modulus less than $K\cong 280$ MeV.

Summarizing the discussion of the results of Ref. 27, we note that the introduction into the theory of nonlinear terms associated with allowance for the self-interaction of the scalar field improves the results of the linear model[66,67] and the agreement with experiment. However, allowance for these new degrees of freedom is not sufficient. In particular, the level density near the Fermi surface is found to be too low, this being a consequence of the small value of the effective nucleon mass in such an approach. Allowance for exchange Fock terms leads to an effective mass that depends on the state, and this can improve the agreement with experiment (see, for example, the studies of Ref. 88, in which calculations were made of the single-particle magnetic moments for nuclei with closed shells plus or minus one nucleon).

We note that in Ref. 89 the Lagrangian (4), (40)

TABLE III. Properties of nuclear matter obtained in the nonlinear model with the two parameter sets of Table II.

Set	$k_F^{(0)}$, fm^{-1}	a_1, MeV	a_2, MeV	a_3, MeV	a_4, MeV	K, MeV	$\mathfrak{M}/M$
I	1,34	15,75	18,5	9	33	280	0,621
II	1,34	15,75	18	7	33	245	0,556

TABLE IV. Fermi surface spectrum for neutrons and protons for the nuclei ^{16}O, ^{40}Ca, ^{48}Ca obtained with the parameter set I. The experimental values are taken from Ref. 87.

Nucleon states	^{16}O				^{40}Ca				^{48}Ca			
	protons		neutrons		protons		neutrons		protons		neutrons	
	theor.	exp.	theor.	exp.	theor.	exp.	theor.	exp.	theor.	exp.	theor.	exp.
$1s_{1/2}$	35,2	40±8	39,1	47	44,8	50±11	52,2	50	49,6	55±9	53,0	
$1p_{3/2}$	17,4	18,4	21,0	21,8	30,3	34±6	37.3	30	36,6	35±7	38,6	
$1p_{1/2}$	12,7	12,1	16,2	15,7	26,8		33,8	27	33,8		36,0	
$1d_{5/2}$					16,1	15,5	22,7	21,9	22,9	20	24,3	16
$2s_{1/2}$					11,2	10,9	17,7	18,2	16,6	15,8	19,5	12,4
$1d_{3/2}$					10,8	8,3	17,3	15,6	18,0	15,7	19,6	12,4
											10,7	9,9

(without allowance for the terms nonlinear in the σ field) was used to investigate the role of the ρ-meson field in the description of the properties of nuclei far from the stability line. These nuclei have neutron number N differing strongly from the proton number Z. For this reason, the isovector vector meson can play a very important part in the considered problem. In Ref. 89, either the approximation of a local density or the relativistic Hartree approximation was used. The dependence of the boundary of quasistable nuclei in the Z, N plane on the parameter $g_\rho^2(M/m_\rho)^2=C_\rho^2$ was investigated. This dependence was found to be very sensitive to the value of C_ρ^2. The authors of Ref. 89 established that the stability line corresponds to the experimental results for $C_\rho^2 \sim 46$.

In Ref. 89 it was also established how the radii of the proton, r_p, and neutron, r_n, distributions in the nucleus change as the number of neutrons is increased or decreased from the value corresponding to the stable nucleus. It was found that the proton radii r_p follow the neutron radii r_n. This type of behavior is insensitive to the value of C.

We also briefly discuss one of the problems of the theory considered in this section. This is the problem of correct allowance for correlation effects. A general scheme for taking them into account was discussed, for example, in Ref. 45, where, in particular, it was noted that exchange effects are automatically contained in the correlation effects, and for this reason they should be taken into account simultaneously. It was also noted in Ref. 45 that the correlation effects are determined by the parameters of the mesons in the nuclear medium. The fact is that the meson polarizes the nuclear medium, and its properties in the medium are not the same as in the vacuum.

Chin[57] noted that in the nuclear medium the scalar and vector mesons become heavier. This effect is most clearly expressed for the π meson. This was demonstrated in Ref. 45. The fact that the mesons are heavier in the nuclear medium could be the reason for the weakening of the correlation effects in nuclei.

In this review, we shall not dwell on the description of the properties of deformed nuclei in the framework of relativistic theory. This description is discussed in detail in Refs. 90 and 91, and also in the review of Ref. 7, where it is shown that models with nonlinear self-interaction reproduce the properties of deformed nuclei; in Ref. 92 the extension of the results obtained for even–even deformed nuclei[90,91] to the case of A-odd nuclei (near nuclei with closed shells) is considered. In Ref. 92, the problem is solved in the mean-field approximation with and without allowance for self-interaction of the scalar field.

The problem of the isoscalar magnetic moments of A-odd nuclei is also considered in Ref. 92. The simplest shell model describes such nuclei in the form of a spherical

TABLE V. Binding energies per particle (in MeV).

Method of determination	^{16}O	^{40}Ca	^{48}Ca	^{90}Zr	^{208}Pb
Experiment	7,98	8,55	8,67	8,71	7,87
Correction for c.m.s. motion	0,61	0,20	0,18	0,08	0,02
Set I	6,68	8,07	8,15	8,55	8,21
Final value Set I	7,29	8,27	8,31	8,63	8,23
Set II	7,14	8,35	8,54	8,84	8,49
Final value Set II	7,85	8,55	8,72	8,92	8,51

core with closed shells, the valence particle determining the magnetic moment of the nucleus. In the framework of such a model, the nonrelativistic theory reproduces the "Schmidt line." However, in the relativistic case, because of the small effective nucleon mass in the medium, the single-particle current of the valence nucleon is increased by a factor $M/\mathfrak{M}$ in relation to the nonrelativistic current, and this leads to a pronounced difference of the calculated isoscalar magnetic moments from the Schmidt values. The absence of agreement between the relativistic predictions and the experimental magnetic moments is related to the appearance of large relativistic potentials in the nucleus. This problem of the magnetic moments has been the subject of many investigations in recent years. The solution of the problem is associated with the fact that the simple shell model gives a poor description of the current corresponding to the self-consistent Hartree ground state of an A-odd nucleus. The fact is that the valence nucleon in such a nucleus is the source of new meson fields, and the response of the nucleons of the core to these fields cannot be ignored in the calculation of the resulting current. In the relativistic case, the result is obtained with compensation of two effects—the enhancement of the current of the valence particle by the low effective nucleon mass and the contribution of the additional current of the polarized nucleons of the core (see Refs. 22–25, 64, and 93–96). This fundamental fact has important consequences for Λ hypernuclei. In this case, the enhancement of the current of the valence hyperon is not completely compensated by the contribution of the core, since the mass of the Λ hyperon and its coupling constant differ from the nucleon values.[97] It should be noted that since the Λ is an isoscalar particle, the results for Λ hypernuclei are relatively free of the uncertainties inherent in isovector currents. Moreover, the single-particle Λ-hyperon current does not contribute to the magnetic moment of the hypernucleus (the Λ is a neutral particle), and therefore the Schmidt value in the hypernucleus is entirely associated with the anomalous moment of the hyperon, and the nonvanishing current of the polarized core is the source of an appreciable deviation from the Schmidt value. These differences of the relativistic and nonrelativistic descriptions can serve as a test for comparing the different approaches.[95,96] At the present time, it is assumed that the study of nuclear currents and, in particular, magnetic moments can provide a possibility for choosing an adequate theory.

Various aspects of the interaction of anitprotons with nuclei in relativistic models were considered in Refs. 98–100.

4. VECTOR DOMINANCE AND THE COULOMB NUCLEAR POTENTIAL

In the nucleus, there is, besides the vector and scalar meson fields, the Coulomb field. As is well known, the nucleons are not point particles, and therefore to construct the Coulomb field of a nucleus it is necessary to take into account the electromagnetic form factors of the nucleons. In Ref. 85, this was done on the basis of Sakurai's vector-dominance model,[101] in the framework of which the nucleons interact with the electromagnetic field through the fields of the ρ and ω mesons (Fig. 4). The Lagrangian of the relativistic shell model with allowance for vector dominance has the form[4)]

$$\mathfrak{L}=\bar{\psi}\gamma^{\mu}(i\partial_{\mu}-g_{\omega}\omega_{\mu}-t_3g_{\rho}\rho_{\mu})\psi-\bar{\psi}(M+g_S\sigma)\psi-\frac{f_{\omega}}{4M}\bar{\psi}\sigma^{\mu\nu}\omega_{\mu,\nu}\psi-t_3\frac{f_{\rho}}{4M}\bar{\psi}\sigma^{\mu\nu}\rho_{\mu,\nu}\psi+\frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma-\frac{1}{2}m_S^2\sigma^2-\frac{1}{4}\omega_{\mu,\nu}\omega^{\mu,\nu}$$
$$+\frac{1}{2}m_{\omega}^2\omega_{\mu}\omega^{\mu}-\frac{1}{4}\rho_{\mu,\nu}\rho^{\mu,\nu}+\frac{1}{2}m_{\rho}^2\rho_{\mu}\rho^{\mu}-\frac{1}{4}A_{\mu,\nu}A^{\mu,\nu}+\frac{1}{2}\left(\frac{e^2m_{\rho}^2}{g_{\rho}^2}+\frac{e^2m_{\omega}^2}{4g_{\omega}^2}\right)A_{\mu}A^{\mu}-\frac{em_{\rho}^2}{g_{\rho}}A_{\mu}\rho^{\mu}-\frac{em_{\omega}^2}{2g_{\omega}}A_{\mu}\omega^{\mu}, \quad (59)$$

where ω_{μ}, ρ_{μ}, A_{μ} are vector fields (meson and electromagnetic); $\omega_{\mu,\nu}=\partial_{\mu}\omega_{\nu}-\partial_{\nu}\omega_{\mu}$ with analogous expressions for $\rho_{\mu,\nu}$ and $A_{\mu,\nu}$; g_{ω}, g_{ρ}, and e are the corresponding coupling constants; and σ and g_S are the scalar field and its coupling constants. For the vector fields, the tensor coupling with the nucleons, $\sigma^{\mu\nu}=(1/2i)(\gamma^{\mu}\gamma^{\nu}-\gamma^{\nu}\gamma^{\mu})$, is also taken into account.[17] It will be shown below that the constants f_{ω} and f_{ρ} can be expressed in terms of the anomalous gyromag-

TABLE VI. Nuclear radii $r_i=\langle r_i\rangle^{1/2}$ (in fm).

Method of determination		^{16}O	^{40}Ca	^{48}Ca	^{90}Zr	^{208}Pb
r_n-r_p	Experiment	0,00	0,03	0,20	0,13	0,16
	Set I	−0,03	−0,05	0,20	0,10	0,23
	Set II	−0,03	−0,05	0,20	0,09	0,23
r_{ch}	Experiment	2,73	3,48	3,47	4,27	5,50
	Set I	2,74	3,45	3,47	4,25	5,49
	Set II	2,75	3,48	3,48	4,27	5,53

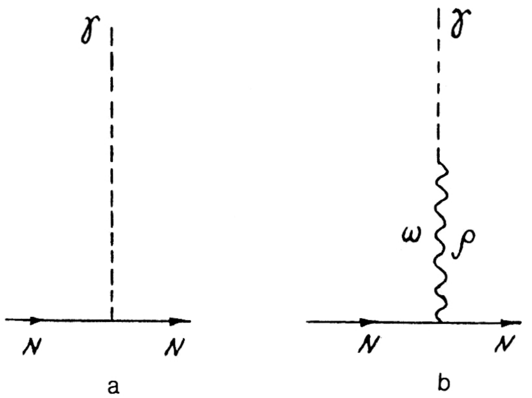

FIG. 4. Interaction of the electromagnetic field with point nucleons (a) and in the vector-dominance model (b).

netic ratios of the nucleons. It is readily verified that the Lagrangian (59) is invariant with respect to the gauge transformation

$$\psi'=e^{-i\left(\frac{1}{2}+t_3\right)\Phi}\psi,\quad \omega'_\mu=\omega_\mu+\frac{1}{g_\omega}\,\partial_\mu\Phi,$$
$$\rho'_\mu=\rho_\mu+\frac{1}{g_\rho}\,\partial_\mu\Phi,\quad A'_\mu=A_\mu+\frac{1}{e}\,\partial_\mu\Phi, \tag{60}$$

where t_3 is the z component of the nucleon isospin, equal to $+1/2$ and $-1/2$ for the protons and neutrons.

By means of the usual Euler–Lagrange scheme, we obtain from (59) the system of coupled equations

$$\left[\gamma^\mu(i\partial_\mu-g_\omega\omega_\mu-t_3g_\rho\rho_\mu)-\frac{f_\omega}{4M}\,\sigma^{\mu\nu}\omega_{\mu,\nu}\right.$$
$$\left.-t_3\,\frac{f_\rho}{4M}\,\sigma^{\mu\nu}\rho_{\mu,\nu}-M-g_S\sigma\right]\psi=0, \tag{61}$$

$$(-\partial_\mu\partial^\mu-m_S^2)\sigma=g_S\bar\psi\psi, \tag{62}$$

$$(-\partial_\mu\partial^\mu-m_\omega^2)\omega^\nu=-g_\omega\bar\psi\gamma^\nu\psi+\frac{f_\omega}{2M}\,\partial_\mu(\bar\psi\sigma^{\mu\nu}\psi)-\frac{em_\omega^2}{2g_\omega}\,A^\nu, \tag{63}$$

$$(-\partial_\mu\partial^\mu-m_\rho^2)\rho^\nu$$
$$=-g_\rho\bar\psi\gamma^\nu t_3\psi+\frac{f_\rho}{2M}\,\partial_\mu(\bar\psi\sigma^{\mu\nu}t_3\psi)-\frac{em_\rho^2}{g_\rho}\,A^\nu, \tag{64}$$

$$\left(-\partial_\mu\partial^\mu-\frac{e^2m_\rho^2}{g_\rho^2}-\frac{e^2m_\omega^2}{4g_\omega^2}\right)A^\nu=-\frac{em_\rho^2}{g_\rho}\,\rho^\nu-\frac{em_\omega^2}{2g_\omega}\,\omega^\nu. \tag{65}$$

Thus, the electromagnetic field has as its source the vector meson fields, one of the sources of which is, in turn, the electromagnetic field.

The relativistic shell model[16,28] is based on replacement of the operators of the nucleon currents on the right-hand sides of Eqs. (62)–(64) by their expectation values in the ground state of the nucleus. Only the time components of the fields ($\nu=0$) are nonzero, since there are no spatial components of the currents in the ground states of even–even nuclei. The fields are static, since in the absence of external fields the nuclear densities do not depend on the time. Therefore, in what follows we shall be interested in static solutions of Eqs. (62)–(65).

In this case, Eqs. (61)–(65) take the form

$$E\psi=\left[\boldsymbol\alpha\cdot\mathbf{p}+g_\omega\cdot\omega_0+t_3g_\rho\rho_0-\frac{i}{2M}\,\gamma(f_\omega\nabla\omega_0\right.$$
$$\left.+t_3f_\rho\nabla\rho_0)+\gamma_0(M+g_S\sigma)\right]\psi, \tag{66}$$

$$(\Delta-m_S^2)\sigma=g_Sn_S\,, \tag{67}$$

$$(\Delta-m_\omega^2)\omega_0=-g_\omega n-\frac{f_\omega}{2M}\,\nabla\mathbf{j}_\sigma-\frac{em_\omega^2}{2g_\omega}\,A_0\,, \tag{68}$$

$$(\Delta-m_\rho^2)\rho_0=-\frac{1}{2}\,g_\rho n_3+\frac{f_\rho}{4M}\,\nabla\mathbf{j}_\sigma^{(-)}-\frac{em_\rho^2}{g_\rho}\,A_0\,, \tag{69}$$

$$\left(\Delta-\frac{e^2m_\rho^2}{g_\rho^2}-\frac{e^2m_\omega^2}{4g_\omega^2}\right)A_0=-\frac{em_\rho^2}{g_\rho}\,\rho_0-\frac{em_\omega^2}{2g_\omega}\,\omega_0\,, \tag{70}$$

where $n=n_n+n_p$ and $n_3=n_p-n_n$ are the isoscalar and isovector density distributions of the nucleons over the nucleus; $\mathbf{j}_\sigma$ and $\mathbf{j}_\sigma^{(-)}$ are the isoscalar and isovector spin currents, determined by the relation

$$\mathbf{j}_\sigma=i\langle\bar\psi\boldsymbol\alpha\psi\rangle \tag{71}$$

(the symbol $\langle\ldots\rangle$ denotes an average over the ground state of the nucleus).

We shall show that the fields ω_0, ρ_0, and A_0 have Coulomb asymptotic behavior. To this end, we rewrite Eqs. (68) and (69) in the form

$$\omega_0=\frac{g_\omega}{m_\omega^2}\,n+\frac{f_\omega}{2Mm_\omega^2}\,\nabla\mathbf{j}_\sigma+\frac{e}{2g_\omega}\,A_0+\frac{1}{m_\omega^2}\,\Delta\omega_0\,, \tag{72}$$

$$\rho_0=\frac{g_\rho}{2m_\rho^2}\,n_3-\frac{f_\rho}{4Mm_\rho^2}\,\nabla\mathbf{j}_\sigma^{(-)}+\frac{e}{g_\rho}\,A_0+\frac{1}{m_\rho^2}\,\Delta\rho_0\,. \tag{73}$$

Substituting (72) and (73) in (70), we obtain

$$\Delta\left(A_0+\frac{e}{g_\rho}\,\rho_0+\frac{e}{2g_\omega}\,\omega_0\right)=-en_p+\frac{ef_\rho}{4Mg_\rho}\,\nabla\mathbf{j}_\sigma^{(-)}$$
$$-\frac{ef_\omega}{4Mg_\omega}\,\nabla\mathbf{j}_\sigma\,. \tag{74}$$

Since $\Delta(1/r)=0$, we see that Eqs. (72)–(74) for $r\gg R$, i.e., far from the nucleus, have the solutions

$$A_0=\frac{a}{r}\,,\quad \omega_0=\frac{ea}{2g_\omega}\,\frac{1}{r}\,,\quad \rho_0=\frac{ea}{g_\rho}\,\frac{1}{r}\,. \tag{75}$$

The constant a can be found by integrating both sides of Eq. (74) over the volume of a sphere whose radius is much greater than the nuclear radius. In this way, we obtain

$$ea=Z\,\frac{e^2}{4\pi}\left(1+\frac{e^2}{g_\rho^2}+\frac{e^2}{4g_\omega^2}\right)^{-1}. \tag{76}$$

Bearing in mind that $g_\rho^2/4\pi=2.575$, $g_\omega^2/4\pi=14.059$ (see below), we obtain

$$\frac{e^2}{g_\rho^2}+\frac{e^2}{4g_\omega^2}=0.003, \tag{77}$$

i.e., the contribution of the $A\rho$ and $A\omega$ interactions to the charge is only 0.3%.

The vector fields of the nucleus can be divided into the Coulomb and nuclear components:

$$\omega_0=\omega_{0C}+\omega_{0N}, \quad \rho_0=\rho_{0C}+\rho_{0N}\,, \tag{78}$$

which are determined by solving the equations

$$\begin{aligned} g_\rho\rho_{0C}&=C+\frac{1}{m_\rho^2}\,\Delta(g_\rho\rho_{0C}),\\ g_\omega\omega_{0C}&=\frac{1}{2}\,C+\frac{1}{m_\omega^2}\,\Delta(g_\omega\omega_{0C}), \end{aligned} \tag{79}$$

where $C=eA_0$ is the Coulomb potential of the nucleus, and

$$(\Delta-m_\rho^2)g_\rho\rho_{0N}=\frac{g_\rho^2}{2}\left(-n_3+\frac{f_\rho}{2Mg_\rho}\,\nabla\mathbf{j}_\sigma^{(-)}\right), \tag{80}$$

$$(\Delta-m_\omega^2)g_\omega\omega_{0N}=-g_\omega^2\left(n+\frac{f_\omega}{2Mg_\omega}\,\nabla\mathbf{j}_\sigma\right). \tag{81}$$

Ignoring the second terms on the right-hand sides of Eqs. (79), we conclude from (66), (78), and (80) that

$$g_\rho\rho_0=C-2U_1\,, \tag{82}$$

i.e., the ρ-meson field of the nucleus is equal to the difference between the Coulomb potential and twice the isovector (U_1) potential. Indeed, from (80) we obtain approximately

$$g_\rho\rho_{0N}=-2U_1(r)\approx\frac{g_\rho^2}{2m_\rho^2}\,n_3(r), \tag{83}$$

in agreement with the phenomenological expression[103] for the nuclear isovector potential U_1.

We now obtain a more accurate expression for the isovector potential that takes into account the finite range of the ρ-meson exchange potential [the terms $\Delta g_\rho\rho_{0N}$ and the divergence on the right-hand side of Eq. (80)]. For this, we substitute the decomposition (78) in Eq. (70) and obtain the approximate expressions $g_\rho\rho_{0C}\approx C$ and $g_\omega\omega_{0C}\approx\frac{1}{2}C$ for the Coulomb components of the vector fields. Then from (70) and (76) we obtain

$$\Delta C=-e^2\left(\frac{m_\omega^2}{2g_\omega}\,\omega_{0N}+\frac{m_\rho^2}{g_\rho}\,\rho_{0N}\right). \tag{84}$$

On the other hand, the source of the Coulomb field is, as is well known, the density $n_C(r)$ of the charge distribution over the nucleus. From this, we obtain an expression for it in terms of the nuclear components of the vector fields,

$$n_C(r)=\frac{m_\rho^2}{g_\rho}\,\rho_{0N}(r)+\frac{m_\omega^2}{2g_\omega}\,\omega_{0N}(r), \tag{85}$$

which is the well-known current–field identity in Sakurai's model.[101]

Ignoring the difference between the forms of $\omega_{0N}(r)$ and $\rho_{0N}(r)$ (this is valid because of the small difference of the masses of the ω and ρ mesons), we conclude that the vector fields of the nucleus are proportional to the charge density distribution $n_C(r)$. In this way, we obtain the more accurate phenomenological expression

$$U_1(r)=K_1\,\frac{N-Z}{Z}\,n_C(r), \quad K_1=339.3\ \mathrm{MeV\cdot fm^3} \tag{86}$$

for the vector potential of the nucleus.

To conclude this section, we consider a nucleon in an external static magnetic field. In this case, Eqs. (63) and (64) give $\omega_\nu=(e/2g_\omega)A_\nu$, $\rho_\nu=(e/g_\rho)A_\nu$, and the Dirac equation (61) takes the form

$$\begin{aligned} E\psi=\Bigg[&\boldsymbol{\alpha}\left(\mathbf{p}-\frac{1+\tau_3}{2}\,e\mathbf{A}\right)+\gamma_0 M-\frac{e}{2M}\,\gamma_0\\ &\times\left(\frac{f_\omega}{2g_\omega}+\tau_3\,\frac{f_\rho}{2g_\rho}\right)\sigma\ \mathrm{curl}\ \mathbf{A}\Bigg]\psi, \end{aligned} \tag{87}$$

where the last term takes into account the anomalous magnetic moments of the nucleons. Using the values 1.79 and −1.91 of the anomalous gyromagnetic ratios of the proton and neutron, we obtain

$$\frac{f_\omega}{2g_\omega}=-0.06;\quad \frac{f_\rho}{2g_\rho}=1.85. \tag{88}$$

Results of calculations

The self-consistent system of equations (66)–(70) determines the properties of the nuclei through the constants g_ω, g_ρ, g_S, and m_S. The calculations made in Ref. 85 of the ground-state properties of the ^{16}O, ^{40}Ca, ^{48}Ca nuclei give for these constants the following "medium" values:

$$\begin{aligned} &\frac{g_\omega^2}{4\pi}=14.059,\quad \frac{g_\rho^2}{4\pi}=2.575,\\ &\frac{g_S^2}{4\pi}=6.81,\quad m_S=475\ \mathrm{MeV}. \end{aligned} \tag{89}$$

The "vacuum" values of these constants are determined by analyzing NN scattering in the model of one-boson exchange. They are

$$\begin{aligned} &\frac{g_\omega^2}{4\pi}=12.85\pm1.29,\quad \frac{g_\rho^2}{4\pi}=2.470\pm0.255,\\ &\frac{g_S^2}{4\pi}=7.14,\quad m_S=550\ \mathrm{MeV}. \end{aligned} \tag{90}$$

Comparing (89) and (90), we see that the "medium" coupling constants of the vector mesons hardly differ from the "vacuum" constants. Thus, the relativistic nuclear shell model proposed in Ref. 85 contains just two parameters (a coupling constant and the mass of the scalar meson) that differ, even if only a little, from the vacuum values.

In Table VII, we give the results of calculations of the mean-square radii of the distributions of the protons, the

TABLE VII. The rms radii of the distributions of the protons, neutrons, and charge (in fm). The experimental data given in the brackets are taken from Refs. 104, 105, and 106.

Nucleus	r_p	r_n	$r_n - r_p$	r_{ch}
^{16}O	2,77	2,72	−0,05 (0,0)	2,83 (2,73)
^{40}Ca	3,43	3,37	−0,06 (0,05±0,05)	3,48 (3,48)
^{48}Ca	3,42	3,62	0,2 (0,2±0,05)	3,44 (3,47)

neutrons, and the charge in the ^{16}O, ^{40}Ca, ^{48}Ca nuclei. The experimental values of these quantities, also given in Table VII, exhibit good agreement with the theory. The charge distributions in these nuclei calculated in accordance with the expression (85) are shown in Fig. 5 by the broken curves. The corresponding experimental data, shown in the same figure by the continuous curves, are taken from the compilation of Ref. 107. It can be seen from the figure that the agreement between the theory and experiment is entirely satisfactory.

For the ^{48}Ca nucleus, the excitation energy of the isobar-analog state was also calculated:

$$\Delta_C = -\frac{g_\rho}{n-Z}\int \rho_{0C}(r) n_3(r) d^3\mathbf{r}.$$

The calculated value $\Delta_C = 7.041$ MeV agrees well with experiment: $\Delta_C^{\exp} = 7.180\,\text{MeV}$.[108]

We see that the quality of the description of the ground-state properties of nuclei in the relativistic shell model is not in the least less good than in the calculations

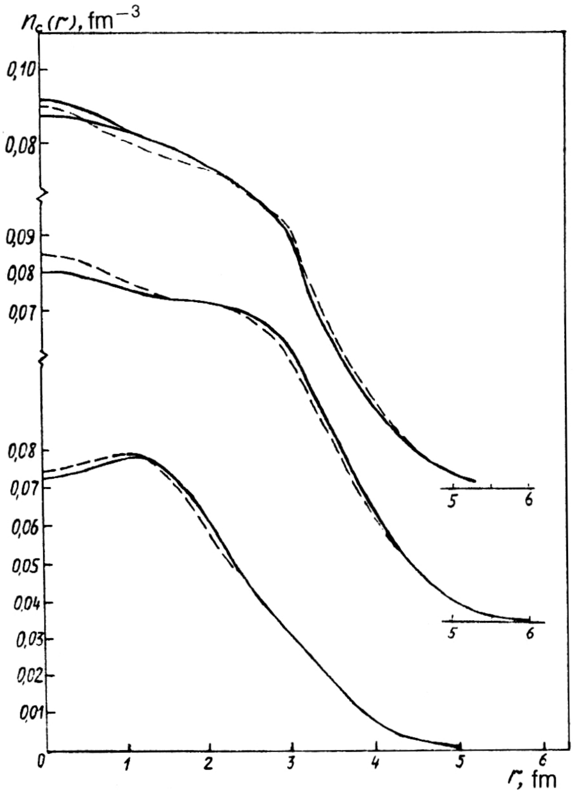

FIG. 5. Charge distribution in the ^{16}O, ^{40}Ca, ^{48}Ca nuclei.

in accordance with the nonrelativistic theory despite the significantly smaller number of phenomenological parameters (the typical number of parameters of the nonrelativistic theory, for example, in the Hartree–Fock–Skyrme method, is 6–7).

Thus, in the framework of Dirac phenomenology it is possible, from a unified point of view and with maximum economy, to describe many phenomena associated with the structure of nuclear ground states. On the one hand, this is a strong argument for using a relativistic treatment in nuclear physics; on the other, it is the reason for an ever increasing stream of papers that both develop further and extend the nuclear phenomena described by the relativistic theory.

5. CONNECTION BETWEEN THE RELATIVISTIC THEORY AND THE HARTREE–FOCK METHOD WITH EFFECTIVE SKYRME FORCES

The results that will be presented below were obtained in the studies of Refs. 14, 45, and 109–112. In this section, we shall adopt the positions of the relativistic phenomenology, in the framework of which the specific nature of the fields in which the nucleons move is not investigated. The development of nuclear theory during recent years indicates that the nucleus is a relativistic system. However, 40 years of the existence of nuclear physics were associated with the development of nonrelativistic representations in nuclear theory. Definite successes were achieved. In particular, the nonrelativistic self-consistent Hartree–Fock theory with effective, density-dependent forces (or three-particle forces),[50] which were introduced by Skyrme [Hartree–Fock–Skyrme (HFS) approach],[49] was widely used. Many investigations were made in this direction, and a reasonable description of the basic characteristics of nuclei was obtained.

In this connection, the following question arises: To what are the successes of the nonrelativistic theory due? Are they due to the use of phenomenological parameters, or is there is a physical reason that allows a corresponding phenomenology? We now turn to a discussion of this question.

We consider the transition from the relativistic Dirac equation (11) to an equation of Skyrme type. We shall calculate the energies of the single-particle states from the nucleon rest mass,

$$E_\lambda = M + \varepsilon_\lambda \tag{91}$$

(for bound states, ε_λ is the binding energy), and we write Eq. (11) as a system of two equations for φ_λ and χ_λ:

$$\varepsilon_\lambda \varphi_\lambda = \boldsymbol{\sigma}\cdot \mathbf{p}\chi_\lambda + [V(r)+S(r)]\varphi_\lambda ,$$
$$\varepsilon_\lambda \chi_\lambda = \boldsymbol{\sigma}\cdot \mathbf{p}\varphi_\lambda - [2M+S(r)-V(r)]\chi_\lambda . \tag{92}$$

We now use the fact that for bound states $\varepsilon_\lambda \ll M$, and therefore in the second equation in (92) the term with ε_λ can be ignored. Then to terms of order ε_λ/M, the system (92) takes the form

$$\chi_\lambda = \frac{1}{2\mathfrak{M}(r)} \boldsymbol{\sigma}\cdot \mathbf{p}\varphi_\lambda , \tag{93}$$

$$\varepsilon_\lambda \varphi_\lambda = \left[\mathbf{p}\frac{1}{2\mathfrak{M}(r)}\mathbf{p} + U(r) + \frac{1}{r}\frac{d}{dr}\left(\frac{1}{2\mathfrak{M}(r)}\right)\mathbf{l}\cdot \boldsymbol{\sigma} \right]\varphi_\lambda , \tag{94}$$

where we have introduced the notation

$$U(r)=V(r)+S(r);\quad 2\mathfrak{M}(r)=2M+S(r)-V(r). \tag{95}$$

Equation (94), which is valid to terms of order ε_λ/M, has the form of the Schrödinger equation for a particle with effective mass $\mathfrak{M}(r)$ (Ref. 102) moving in the central potential $U(r)$ and a spin–orbit potential. The HFS method leads to equations of precisely such form.[50,113]

The HFS method is based on the effective forces introduced by Skyrme.[49] Vautherin and Brink[50] used as their point of departure a Skyrme potential in the form of a sum of two-particle and three-particle forces,

$$\mathfrak{U} = \sum_{i<j} v_{ij}^{(2)} + \sum_{i<j<k} v_{ijk}^{(3)}, \tag{96}$$

employing as interactions $v_{12}^{(2)}$ and $v_{123}^{(3)}$ in the configuration space velocity-dependent zero-range interactions of the form

$$\begin{aligned} v_{12}^{(2)} &= t_0(1+x_0P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2) + \tfrac{1}{2}t_1[\delta(\mathbf{r}_1-\mathbf{r}_2)k^2 \\ &\quad + k'^2\delta(\mathbf{r}_1-\mathbf{r}_2)] + t_2\mathbf{k}'\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k} + iW_0 \\ &\quad \times(\sigma_1+\sigma_2)\mathbf{k}'\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k} \end{aligned} \tag{97}$$

and

$$v_{123}^{(3)} = t_3\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_2-\mathbf{r}_3), \tag{98}$$

where t_0, t_1, t_2, t_3, x_0, W_0 are parameters; P_σ is the operator of spin exchange; $\mathbf{k}$ denotes the operator $(1/2i)(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2)$ acting to the right, and $\mathbf{k}'$ is the operator $-(1/2i)(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2)$ acting to the left; the last term in (97) represents the two-particle spin–orbit forces. In Hartree–Fock calculations of even–even nuclei, the three-particle forces (98) are equivalent to repulsive two-particle forces with a linear dependence on the density of the form (26), and they ensure saturation in the HFS method.

The Hartree–Fock equations for the Skyrme interaction can be obtained[50] by requiring stationarity of the total energy of the nucleus with respect to variations of the single-particle states φ_λ and by imposing the additional requirement of normalization of these states.

For the Skyrme interaction, the energy density is[50] an algebraic function of the nucleon densities n_n and n_p, the kinetic-energy densities τ_n and τ_p, and also the spin densities $\mathbf{J}_n$ and $\mathbf{J}_p$, which are determined by means of the relations

$$n(\mathbf{r}) = \sum_{\lambda,\sigma} |\varphi_\lambda(\mathbf{r},\sigma)|^2,\quad \tau(\mathbf{r}) = \sum_{\lambda,\sigma} |\boldsymbol{\nabla}\varphi_\lambda(\mathbf{r},\sigma)|^2,$$
$$\mathbf{J}(\mathbf{r}) = -i\sum_{\lambda,\sigma,\sigma'} \varphi_\lambda^*(\mathbf{r},\sigma)[\boldsymbol{\nabla}\varphi_\lambda(\mathbf{r},\sigma')\times\langle\sigma|\boldsymbol{\sigma}|\sigma'\rangle]. \tag{99}$$

The sums in Eqs. (99) are calculated over all occupied single-particle states: σ, $\sigma'=\pm 1/2$.

For nuclei with $N=Z$ and without a Coulomb field, we have

$$n_n=n_p=\tfrac{1}{2}n,\quad \tau_n=\tau_p=\tfrac{1}{2}\tau,\quad \mathbf{J}_n=\mathbf{J}_p=\tfrac{1}{2}\mathbf{J}. \tag{100}$$

For spin-saturated nuclei, the spin density vanishes (this last assertion is valid to the accuracy with which the radial wave functions of the spin–orbit doublet $j=l\mp 1/2$ can be assumed to be identical):

$$\mathbf{J}=0. \tag{101}$$

If, for simplicity, we restrict consideration to nuclei for which the conditions (100) and (101) are satisfied, then in this case we obtain as Hartree–Fock equations for the Skyrme potential (97), (98) equations of the form[50]

$$\varepsilon_\lambda\varphi_\lambda = \left[\mathbf{p}\frac{1}{2\mathfrak{M}^*(r)}\mathbf{p} + U_{\rm HFS} + \mathbf{W}(\mathbf{r})(-i)(\boldsymbol{\nabla}\times\boldsymbol{\sigma})\right]\varphi_\lambda , \tag{102}$$

where $\mathfrak{M}^*(r)$ is the effective mass; $U_{\rm HFS}$ is the potential energy; and $\mathbf{W}(\mathbf{r})$ is the form factor of the single-particle spin–orbit potential in the HFS method. These quantities can be calculated by means of the relations

$$\frac{1}{2\mathfrak{M}^*} = \frac{1}{2M} + \alpha n(r), \tag{103}$$

$$U_{\rm HFS} = -an + bn^2 - c\Delta n + e\tau, \tag{104}$$

$$\mathbf{W}(\mathbf{r}) = \frac{3}{4}W_0\boldsymbol{\nabla}n, \tag{105}$$

where Δ is the Laplacian.

The constants α, a, b, c, e in Eqs. (103)–(105) can be expressed in terms of the parameters t_0, t_1, t_2, and t_3 of the Skyrme forces.

Comparing Eq. (94) of the relativistic theory for the large component of the wave function with Eq. (102) of the HFS method, we see that these equations have much in common.

The difference between these equations is that in the HFS method the spin–orbit potential is introduced "by hand," whereas in the relativistic theory it has the same origin as the effective mass. We note also one further feature.

In the HFS method, we have the equality $\alpha=e$; as we shall see in what follows, such a relation is not satisfied in the relativistic theory.

We begin our consideration only with Eq. (11) and estimate, using the single-particle equation (94) of Skyrme type, the values of the fields $V(r)$ and $S(r)$ on the basis of experimental facts—the known values of the spin–orbit coupling and the depth of the shell potential in the nucleus.

The single-particle spin–orbit potential in the nucleus [see Eq. (94)],

$$U_{LS}=\frac{1}{r}\frac{d}{dr}\left(\frac{1}{2\mathfrak{M}(r)}\right)\mathbf{1}\cdot\boldsymbol{\sigma}, \tag{106}$$

can be transformed to the form

$$U_{LS}=\frac{1}{r}\frac{[V(r)-S(r)]'}{(2\mathfrak{M})^2}\mathbf{1}\cdot\boldsymbol{\sigma}, \tag{107}$$

from which it follows that the enhancement of the spin–orbit coupling in the nucleus arises for two reasons: 1) because $|V-S|\gg|U|$; 2) because $\mathfrak{M}<M$. The expression (106) ensures the correct sign and magnitude of the spin–orbit interaction in the nucleus.

For an estimate, we use the fact that the spin–orbit potential is known experimentally. Usually, it is chosen in the form

$$U_{LS}=\frac{\alpha_{LS}}{r}\frac{dn}{dr}\mathbf{1}\cdot\boldsymbol{\sigma}, \tag{108}$$

where α_{LS} is a known constant.

Comparing (106) and (108), we obtain

$$\frac{1}{2\mathfrak{M}(r)}=\frac{1}{2M}+\alpha_{LS}n(r), \tag{109}$$

and this is the expression for the effective mass to which the HFS method leads [see (103)].[5)]

In deriving the relation (109), we took into account the boundary conditions

$$n(r)=0,\quad \mathfrak{M}(r)=M \quad \text{for} \quad r>R. \tag{110}$$

It follows from (93) and (109) that the small component of the nucleon wave function can be expressed in terms of its large component by means of the spin–orbit coupling constant α_{LS} [see (108)]:

$$\chi_\lambda=\frac{1}{2M}[1+2M\alpha_{LS}n(r)]\boldsymbol{\sigma}\cdot\mathbf{p}\varphi_\lambda. \tag{111}$$

Substituting $\alpha_{LS}=85.5\ \text{MeV}\cdot\text{fm}^5$ in (109) and (111), and assuming within the nucleus $n=0.17\ \text{fm}^{-3}$, we obtain

$$2M\alpha_{LS}n(r)=0.7 \quad \text{for} \quad r<R, \tag{112}$$

from which it can be seen that the second term in (111) is comparable with the first. The theory of the electromagnetic interaction in the nucleus is based on allowance for only the first term in (111). This means that the use of the nonrelativistic description may involve a large error. This error is manifested, in the first place, in a consideration of transitions of magnetic type, since in this case the large and small components of the wave function are coupled. This was illustrated in the studies of Ref. 45 in a consideration of the problem of the stability of the ground state of a system in the relativistic theory.

Further, from (109) and (112) we have

$$\frac{\mathfrak{M}}{M}=0.6. \tag{113}$$

Hence and from (95) we find that within the nucleus

$$V-S=0.8M\approx 750\ \text{MeV}. \tag{114}$$

Note that in (114) and in what follows the depths of the potentials $S(r)$ and $V(r)$ are denoted by S and V, respectively; U is the depth of $U(r)$. The depth of the central potential can be estimated on the basis of the fact that the power of the well is determined by the expression $2\mathfrak{M}UR^2$ (where R is the radius of the nucleus) and that the depth of the shell potential is -50 MeV. Bearing in mind that in the shell model $\mathfrak{M}=M$, we have

$$U=S+V=\frac{M}{\mathfrak{M}}U_{\text{sh}}\cong-90\ \text{MeV},$$

whence

$$S\cong-420\ \text{MeV};\quad V\cong+330\ \text{MeV}, \tag{115}$$

in agreement with the estimates of Ref. 114, which were made in the framework of the shell model; in this section, we obtained the estimate (115) using the equation of Skyrme type (94) for the large component of the wave function φ_λ.

Calculations in accordance with the expressions (16) and (17) for nuclear matter on the basis of Walecka's model[16] give $S=-410$ MeV and $V=+324$ MeV.

We note that the estimate (115) obtained using the relativistic phenomenology is reliable, since it does not use any model ideas about the nature of the fields $S(r)$ and $V(r)$. This estimate is based solely on experimental data.

Thus, a specific feature of the nucleus is that the fields which act on a nucleon within the nucleus are not small compared with its rest mass. This is why the nucleus is a relativistic system even though $\varepsilon_\lambda/M\ll 1$. The presence of the small parameter ε_λ/M makes it possible to use the quasirelativistic equation (94) to calculate the large components of the wave functions. However, as we have already noted, the small components of the wave functions are strongly enhanced, so that the neglect of them can lead to incorrect physical results.

We demonstrate this for the example of the description of saturation in the theory. It can be seen from (12), (13), and (95) that the depth of the central potential is determined by the expression

$$U=\frac{g_\omega^2}{m_\omega^2}n-\frac{g_S^2}{m_S^2}n_S. \tag{116}$$

We have used the fact that within the nucleus, i.e., for $r<R$, the densities are constant to a good accuracy. Therefore, within the nucleus we can use the results obtained for nuclear matter (see Ref. 45). Figure 6 shows the depth U as a function of n, calculated with the parameters

$$M^2\frac{g_\omega^2}{m_\omega^2}=195.7,\quad M^2\frac{g_S^2}{m_S^2}=266.9. \tag{117}$$

The presence of the minimum of U at a certain value of the density arises because the scalar density n_S is equal to n at low densities, after which it becomes constant (see Fig. 1; Ref. 45). It is the presence of this minimum that ensures saturation. The fact is that in the considered scheme the kinetic energy is a monotonically increasing function of the density. Therefore, if the potential energy has a minimum at a certain density n_{eq}, then the total energy will also have a minimum, i.e., saturation will be achieved, but this minimum will occur at a density $n_0 < n_{eq}$. It is interesting to note that the equilibrium density corresponding to the minimum of the potential energy has the value $n_{eq}=0.216\ \mathrm{fm}^{-3}$ (Ref. 14), whereas the equilibrium density calculated in Ref. 16 from the total energy for the same parameters is $n_0=0.190\ \mathrm{fm}^{-3}$. From these numbers it can be seen that in the considered scheme the kinetic energy plays a small part in ensuring the saturation phenomenon.

In the nonrelativistic approximation, the small components of the wave functions are not taken into account, and as a consequence of this $n_S=n$, so that the depth of the central potential is

$$U_1=-\left(\frac{g_S^2}{m_S^2}-\frac{g_\omega^2}{m_\omega^2}\right)n=-625.68n, \qquad (118)$$

i.e., it does not have any minimum as a function of the density.

From the above treatment it can be seen that saturation, a fundamental property of nuclei, is a purely relativistic effect. This property is realized because there are in a nucleus two fields with the transformation properties of a relativistic scalar and a relativistic vector; also important is the presence of the "small" component of the nucleon wave function. In nonrelativistic theory, we can only imitate this. To ensure in the nonrelativistic theory the existence of a minimum of U at the same value of the density n_{eq} and at the same depth as in the original relativistic potential (116), we can use a quadratic form like

$$U_{II}=-an+bn^2, \qquad (119)$$

which corresponds to the HFS method [see Eq. (104)]. The potential U_{II} has a minimum at the same point as the potential (116), at

$$a=816\ \mathrm{MeV}\cdot\mathrm{fm}^3;\quad b=1889\ \mathrm{MeV}\cdot\mathrm{fm}^6 \qquad (120)$$

(see Fig. 6). From this it can be seen directly that the transition from realistic forces to effective forces does not come about as a result of renormalization by many-particle effects, as is assumed in nonrelativistic theory, but by the need to compensate for the use of an inadequate approach, which eliminates from consideration the small component of the nucleon wave function. It should be emphasized that the relativistic potential U does not directly contain a component proportional to n^2. The minimum of the relativistic potential (as a function of the density) is achieved only by virtue of allowance for the small component of the nucleon wave function.

The above considerations show that the nonrelativistic Hartree–Fock calculations with effective Skyrme forces are an imitation of the relativistic description (cf., however, Ref. 110).

The expression (116) for U is valid only within the nucleus. Allowance for the finite sizes leads to a correction of the form

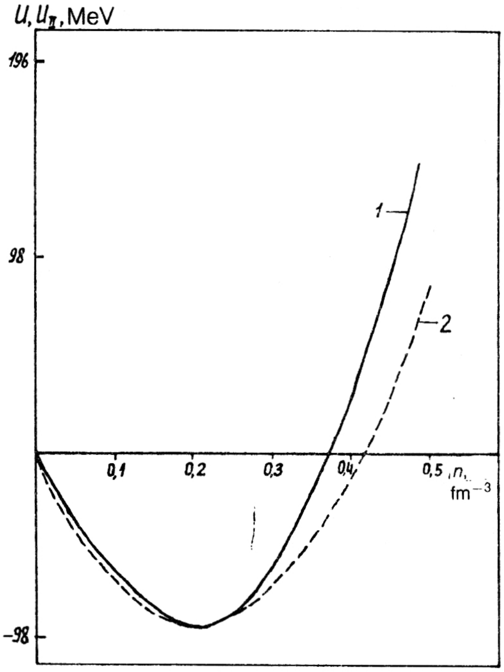

FIG. 6. Dependence of the potentials U (curve 1) and U_{II} (curve 2) on the density n.[14]

$$\delta_1 U=\frac{g_\omega^2}{m_\omega^4}\,\Delta n-\frac{g_S^2}{m_S^4}\,\Delta n_S\approx-\left(\frac{g_S^2}{m_S^4}-\frac{g_\omega^2}{m_\omega^4}\right)\Delta n. \qquad (121)$$

In the HFS method, the central potential (104) also has a term of the form (121). The coefficient of Δn, calculated with realistic forces, is $200\ \mathrm{MeV}\cdot\mathrm{fm}^5$ for $m_S=550$ MeV and $143\ \mathrm{MeV}\cdot\mathrm{fm}^5$ for $m_S=500$ MeV.

Finally, as can be seen from (12), (13), (94), (95), and (116), the central potential has the term

$$\delta_2 U=\left(\frac{1}{2\mathfrak{M}}\right)^2\left(\frac{g_\omega^2}{m_\omega^2}+\frac{g_S^2}{m_S^2}\right)\tau=125\tau, \qquad (122)$$

where τ is the kinetic-energy density, $\tau=\Sigma_\lambda|\nabla\varphi_\lambda|^2$, and such a term is present in the HFS method [see (104)].

Thus, we have shown that the relativistic theory makes it possible to calculate all the parameters of the effective Skyrme forces on the basis of realistic forces. The parameters that we have calculated are compared with the phenomenological values used in various parametrizations in Table VIII, from which it can be seen that the "realistic" parameters agree well with the phenomenology.

There exist several forms of Skyrme forces that describe the properties of nuclear ground states approximately equally well. The results show that the "Skyrme II" forces are the best. The relativistic theory provides a justification of this parametrization of the effective forces.

Thus, it has been shown that the large component of the wave function can be found approximately as the solution of the nonrelativistic problem and that the small component can be recovered from the observed spin–orbit splitting by means of the relation (111). In the description of fields of electric type, the small component can be ig-

TABLE VIII. Comparison of parameters of Skyrme forces and forces calculated on the basis of meson theory.

Coefficient / Type of Skyrme forces[113]	a, MeV · fm^5	a, MeV · fm^3	b, MeV · fm^6	c, MeV · fm^5	e, MeV · fm^5
SI	13	793	2712	82	13
SII	101	877	1750	169	101
SIII	44	846	2625	126	44
SIV	154	904	938	210	154
SV	215	936	0	256	215
SVI	7.7	826	3188	98	7.7
Realistic forces	85.5	816	1889	200	125

nored in the long-wavelength approximation. However, allowance for it is of fundamental importance when one is considering fields of magnetic type, since in this case the interaction couples the large and small components, and the effect of this coupling is enhanced by the decrease of the nucleon effective mass within the nucleus. Such effects can be very important in investigating the stability of the ground state in external fields; fields whose symmetry differs from that of the ground state are of the greatest interest.

In this section, we have not hitherto made a special separation of the Coulomb energy in the single-particle Dirac equation.

To conclude this section, we mention results obtained in Ref. 112, in which a study was made of the Dirac equation for a proton in a nucleus with a shell potential including nuclear and Coulomb components. The reduction of the Dirac equation to an equation of nonrelativistic type was considered in this study. In this case, the effective potential $U_{\rm eff}$ includes two effects associated with the Coulomb interaction that are absent in the traditional nonrelativistic treatment: 1) $U_{\rm eff}$ includes a Coulomb–nucleus interference term; 2) $U_{\rm eff}$ depends strongly on the proton energy, which, in its turn, depends on the Coulomb energy. If the nuclear potential of the shell model consists of a strong attractive world scalar and a strong repulsive Lorentz vector, then the effect (1) is by itself very important, although the effect (2) works in the opposite direction.

The upshot is a small but nevertheless important decrease of the Coulomb energy in relation to the completely nonrelativistic problem.

The relativistic mean-field approximation in its modern form completely ignores the important relativistic effect associated with allowance for the states with negative energy. In this approximation, only the valence nucleons contribute to the sources of the meson fields. In fact, the nucleons in the sea of negative energies are under the strong influence of these mean fields, and this leads to vacuum polarization in such theories. The reason for the neglect of such an effect in the relativistic mean-field approximation is obvious—the Dirac sea contains an infinite number of levels. An attempt to solve this problem for nuclear matter was made in Refs. 57, 115, and 116, which established a significant role of the considered effects, leading to a need to readjust the model parameters. For finite nuclei, the solution to the problem is much more complicated. The problem was considered, for example, in Ref. 117, which also gives references to preceding studies, both for spherical and for deformed nuclei.

6. RELATIVISTIC HARTREE–FOCK AND BRUECKNER–HARTREE–FOCK METHODS FOR SPHERICAL NUCLEI

Whereas the literature contains many studies with calculations of finite nuclei based on the relativistic Hartree method, relativistic Hartree–Fock calculations are much less common. It is important to investigate the role of the Fock terms, since they are essentially associated with isovector mesons. Indeed, as we noted above, the isovector mesons do not contribute at all to the Hartree terms for $N=Z$ systems. Even in the case of $N\neq Z$ systems, the π meson, whose role might seem to be important, does not contribute to the Hartree approximation. The main interest in the Hartree–Fock model, which is more complicated in its practical realization, is associated with the possibility of taking into account in this case the isovector π and ρ mesons (in the case of $N=Z$ nuclei too).

If we proceed from a Lagrangian with only isoscalar (σ, ω) mesons, then in the framework of the Hartree–Fock approximation we obtain results analogous to the Hartree results with a new fitting of the coupling constants (g_σ, g_ω).[118] This result can be readily understood qualitatively if it is recalled that the NN interaction induced by the heavy σ and ω mesons has short range. In the zero-range approximation, the effects of the Fock (exchange) terms are proportional to the corresponding Hartree (direct) contributions.

In connection with the inclusion of the π meson in the self-consistent scheme, we note that the use in the Lagrangian of pseudoscalar coupling leads to computational instabilities in the solution of the Hartree–Fock equations, and for this reason the calculations were made using pseudovector coupling.

In contrast to the Hartree approximation, in the framework of which the corrections associated with the vacuum fluctuations were included in the treatment for finite nuclei,[115–117,119] it is much more complicated to take into account these corrections in the Hartree–Fock approximation.[120] This remains an open problem.

A large proportion of the Hartree–Fock calculations for finite nuclei were made with allowance for only the vector coupling of the ρ meson (Refs. 30, 32, 34, and 121). To a certain degree, this restriction simplifies the calculation of the Fock terms. However, one must have a critical attitude to the restriction, since for vacuum the tensor ρ–N coupling constant f_ρ is much greater than the vector constant g_ρ. The ratio f_ρ/g_ρ is 3.7 in the vector-dominance model, and analysis of πN scattering gives 6.6.

The results of Refs. 30, 32, 34, and 121 show that in the framework of the relativistic Hartree–Fock method it is possible to obtain a description of finite nuclei like the Hartree description. The inclusion of the π meson and the contribution of the exchange terms improves the spin–orbit splittings compared with the Hartree approximation.

Hartree–Fock calculations of finite nuclei based on a Lagrangian including the σ, ω, ρ, and π mesons and allowing both vector and tensor coupling for the ρ meson were made in Ref. 118. In this study, the adjustable parameters were the coupling constants for the σ and ω mesons and the mass of the σ meson; the coupling constants for the π and ρ mesons were taken in Ref. 118 equal to their vacuum values: $f_\pi^2/4\pi=0.08$, $g_\rho^2/4\pi=2.20$, $f_\rho/g_\rho=3.7$, as were the masses of the ω, π, and ρ mesons. The coupling constants g_σ and g_ω were fixed by fitting the saturation point for symmetric nuclear matter ($E/A=-15.75$ MeV, $k_F=1.30\ \mathrm{fm}^{-1}$). The remaining parameter m_σ was taken equal to 440 MeV in order to reproduce the correct ^{16}O radius. With such a Lagrangian, the coefficient a_4 of the symmetry energy obtained in the framework of the Hartree–Fock approximation is 28 MeV, i.e., in acceptable agreement with the empirical value. This result is interesting, since the symmetry energy is to a large degree associated with the isovector mesons, the coupling constants of which are not changed from the vacuum values. It should be noted that the compression modulus K was then found to be 465 MeV, i.e., larger than the empirical value 210 MeV, but nevertheless smaller than the value 540 MeV corresponding to the traditional σ–ω model and the Hartree approximation.[1]

The characteristic results of Ref. 118 for nuclei with closed shells are shown in Fig. 7 and in Table IX.

The charge densities calculated by the Hartree–Fock method are compared with the measured densities in Fig. 7. The level of agreement for all nuclei is fully comparable with that obtained in the nonrelativistic Hartree–Fock calculations with Skyrme forces.[113] The binding energies, charge radii, and some spin–orbit splittings are given in Table IX. Since the corrections for the center-of-mass motion cannot readily be included in the relativistic calculations by the Hartree–Fock method, an estimate taken from nonrelativistic calculations[113] is given in the bottom (c.m.) row of Table IX. The role of these corrections reduces to an increase of the calculated binding energies. Nevertheless, it can be seen that for the binding energies of all nuclei the values fall short of the experimental values by ~ 1.5 MeV per nucleon. It is possible that correlation effects,

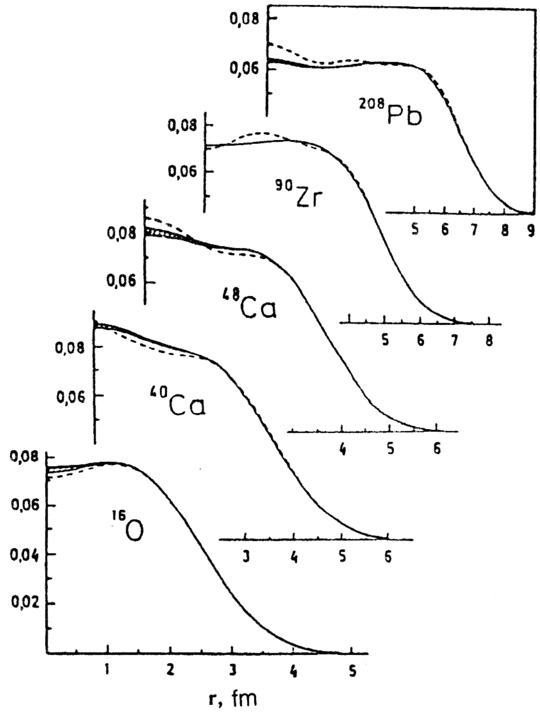

FIG. 7. Calculated (broken curves) and experimental (continuous curves) charge distributions. The hatched regions correspond to the experimental uncertainties.

which are outside the framework of the Hartree–Fock method, will increase the theoretical binding energies. It can also be seen that in the calculations completely satisfactory charge radii of all nuclei are obtained if the value $m_\sigma=440$ MeV is chosen in order to reproduce the ^{16}O charge radius. Although the Hartree–Fock single-particle spectra differ from the experimental spectra, as they must, nevertheless it must be justified to make a direct comparison of the calculated and measured spin–orbit splittings, since the effects associated with going beyond the framework of the Hartree–Fock method lead to a modification of the single-particle energies that evidently preserves the spin–orbit splittings. Table IX illustrates the remarkable agreement between the theory and experiment for Δ_{LS}. Here, the role of the π and ρ mesons is extremely important in order to obtain the correct spin–orbit splittings.

We now establish the relationship between the nuclear binding energies in the relativistic and nonrelativistic calculations. We shall follow the discussion given in Ref. 118. In the relativistic case, the expression for the binding energy per particle has the form

$$\frac{E^R}{A}=\frac{1}{2}\sum_{\alpha=1}^{A}\frac{t_\alpha^R}{A}+\frac{1}{2}\sum_{\alpha=1}^{A}\frac{\varepsilon_\alpha^R}{A}\equiv t^R+\varepsilon^R. \tag{123}$$

In the nonrelativistic Hartree–Fock method, this energy is determined by the relation

$$\frac{E^{NR}}{A}=\frac{1}{2}\sum_{\alpha=1}^{A}\frac{t_\alpha^{NR}}{A}+\frac{1}{2}\sum_{\alpha=1}^{A}\frac{\varepsilon_\alpha^{NR}}{A}+E_r\equiv t^{NR}+\varepsilon^{NR}+E_r, \tag{124}$$

TABLE IX. Comparison of relativistic Hartree–Fock and experimental values. For each nucleus, the table gives the binding energy per particle, the charge radius, and some proton spin–orbit splittings ($1p$ state for ^{16}O, $1d$ state fo. ^{40}Ca and ^{48}Ca). The row c.m. has a commentary in the text. The energies are measured in mega-electron-volts, and the radii in fermis.

Method of determination	^{16}O			^{40}Ca			^{48}Ca			^{90}Zr		^{208}Pb	
	$-\frac{E}{A}$	r_C	Δ_{LS}	$-\frac{E}{A}$	r_C	Δ_{LS}	$-\frac{E}{A}$	r_C	Δ_{LS}	$-\frac{E}{A}$	r_C	$-\frac{E}{A}$	r_C
DHF	5,61	2,73	7,3	6,82	3,47	8,0	7,10	3,47	4,1	7,40	4,26	6,74	5,47
Exp.	7,98	2,73	6,3	8,55	3,48	7,2	8,67	3,47	4,3	8,71	4,27	7,87	5,50
c.m.	0,61			0,20			0,18			0,08		0,02	

where t_α^{NR} is calculated by means of the operator $-\nabla^2/2M$, and E_r is the energy of the rearrangement per particle associated with the density dependence of the effective interaction.

The nonrelativistic wave functions are well approximated by the large components G_α of the relativistic functions, and it can be shown[29] that t^R is less than t^{NR}. To simplify the arguments, we consider the (σ, ω) model, which contains only the scalar field $S(\mathbf{r})$ and the time component of the vector field $V(\mathbf{r})$. In this model, t^R can be expressed in terms of the small components F_α of the wave function as follows:

$$t^R = \sum_{\alpha=1}^{A} \int F_\alpha^2(\mathbf{r})[E_\alpha + S(\mathbf{r}) - V(\mathbf{r})]d\mathbf{r}. \qquad (125)$$

On the other hand, if we calculate t^{NR} using as wave functions only G_α, i.e., to terms of order F_α^2/G_α^2 by virtue of the normalization, and express the results in terms of F_α by means of the Hartree–Dirac equation, then we can obtain

$$t^{NR} = \frac{1}{4M} \sum_{\alpha=1}^{A} \int F_\alpha^2(\mathbf{r})[E_\alpha + M + S(\mathbf{r}) - V(\mathbf{r})]^2 d\mathbf{r}. \qquad (126)$$

Thus,

$$t^R - t^{NR} = -\frac{1}{4M} \sum_{\alpha=1}^{A} \int F_\alpha^2(\mathbf{r})[E_\alpha - M + S(\mathbf{r}) - V(\mathbf{r})]^2 d\mathbf{r} \qquad (127)$$

is always negative.

It can be seen from this result that if the two models give approximately the same binding energies and densities, then ε^{NR} must be more negative than ε^R and/or E_r must make a negative contribution. As an illustration, in Table X we compare the components of the binding energy per particle for the ^{40}Ca nucleus in two realistic cases. The relativistic Hartree–Fock model is considered in Ref. 118, whereas the nonrelativistic model is calculated with a standard Skyrme interaction (SIII)[113] without corrections for the center-of-mass motion in order to make the comparison clearer. Although E^R/A is larger than E^{NR}/A, both t^R and ε^R are lower than t^{NR} and ε^{NR}, respectively. If by fitting (for example, by a small increase of f_ρ) we require $E^R/A = -8.3$ MeV in ^{40}Ca, then t^R and ε^R will be even smaller.

We now briefly discuss Dirac–Brueckner–Hartree–Fock (DBHF) calculations for finite nuclei. Strictly speaking, such calculations have not yet been made on account of the great volume and complexity of the calculations associated with this method. For infinite nuclear matter, the problem of the double self-consistency of the DBHF equations was solved by several groups.[13,122,123] Starting from a quasipotential V obtained on the basis of one-boson exchange potentials (OBEP) with parameters fixed by fitting the NN data, the medium NN interaction (or G matrix) can be found as the solution of the Bethe–Goldstone equation

$$G = V + VKKQG, \qquad (128)$$

where Q is the Pauli operator, and K is the propagator of the dressed particle, satisfying the Dyson equation

$$K = K^{(0)} + K^{(0)}\Sigma K. \qquad (129)$$

In Eq. (129), $K^{(0)}$ is the free-particle propagator, and Σ is the self-energy operator, the matrix elements of which are given by the sum of the Hartree and Fock diagrams:

TABLE X. Components of binding energy per particle for the ^{40}Ca nucleus calculated in the nonrelativistic model NR(SIII) and the relativistic model (R).[118] All quantities are measured in mega-electron-volts.

Parameters	NR(SIII)	R
t	8,2	5,8
ε	−10,9	−12,6
E_r	−5,6	0
E/A	−8,3	−6,8

TABLE XI. Neutron, proton, and charge rms radii calculated in different approximations. The final column gives the binding energies per particle.

Nucleus	Approximation	R_n, fm	R_p, fm	R_c, fm	E/A, MeV
^{40}Ca	1	3,08	3,12	3,22	−9,5
	2	3,21	3,26	3,35	−6,6
	3	3,19	3,23	3,33	−7,9
	exp.			3,48	−8,55
^{90}Zr	1	4,07	4,03	4,11	−8,5
	2	4,08	4,03	4,11	−6,9
	3	4,07	4,03	4,11	−7,9
	exp.			4,27	−8,71
^{208}Pb	1	5,42	5,37	5,43	−7,0
	2	5,36	5,27	5,33	−6,0
	exp.			5,50	−7,87

$$\Sigma(k)=\sum_{k'(occ)}\langle kk'|G|kk'\rangle-\langle kk'|G|k'k\rangle. \tag{130}$$

The self-consistent solution of Eqs. (128)–(130) shows that if we work in the Dirac (four-component) scheme, instead of in the traditional nonrelativistic one, then the saturation point of nuclear matter is shifted from the so-called Coester line and approaches the empirical saturation point (Refs. 2, 13, 122, and 123). At the same time, the compression modulus of nuclear matter reaches values close to the empirical one, i.e., $\sim$200 MeV.[13]

Obviously, it is difficult to obtain a complete solution of Eqs. (128)–(130) for finite nuclei, and for this reason only approximate calculations have so far been made in this case. In Ref. 124, the effective G matrix is obtained from calculations of nuclear matter using a certain version of the effective-density approximation. This effective G matrix has been used to solve Eqs. (129) and (130) for the ^{16}O nucleus. On the basis of the OBEP potentials of Brockmann and Machleidt,[12] Müther, Machleidt, and Brockmann[124] obtained for the binding energy the value 5.65 MeV (the experimental value is 7.98 MeV) and for the charge radius 2.46 fm (experimentally, 2.73 fm). They also compared their results with an analogous calculation made nonrelativistically (on the basis of the same original OBE potential), which gives for the binding energy the value 5.71 MeV, which differs little from the relativistic value, whereas the radius is much less than the relativistic value 2.29 fm.

A different approach was realized in Ref. 125, in which an attempt was made to obtain for a finite system the self-energy Σ (or, at least, its main part) directly from the self-energy Σ^{NM} calculated for nuclear matter using the local-density approximation. The study of Ref. 125 was based on the results of ter Haar and Malfliet[13] for nuclear matter, for which an effective density-dependent Lagrangian was constructed.[126] The main assumption made was that the self-energy in the Dirac equation, which determines the Hartree–Fock spinors, is a sum of a local operator and a correction of the form

$$\Sigma(\mathbf{r})=\Sigma_{\text{bulk}}(\mathbf{r})+\Delta\Sigma. \tag{131}$$

The self-energy for nucler matter, $\Sigma^{NM}(k,\ k_F)$, depends on the nucleon momentum k and the Fermi momentum k_F. By virtue of the weak dependence on k, it is possible to identify Σ_{bulk} with

$$\Sigma_{\text{bulk}}(\mathbf{r})=\Sigma^{NM}[k=k_F,\ k_F(\mathbf{r})], \tag{132}$$

where the appearance of $k_F(\mathbf{r})$ reflects the local-density approximation. The term $\Delta\Sigma$ in (131) represents a certain correction for the finite radius, since it vanishes for a G matrix of zero range.

Various approximations were used for $\Delta\Sigma$:

1) The correction $\Delta\Sigma$ was set equal to zero. In this case, the dependence in (132) on $k_F(\mathbf{r})$ is replaced by a dependence on the averaged local momentum $k_F(\mathbf{r})$.

2) It was assumed that

$$\Delta\Sigma=\Sigma_H(\mathbf{r})-\Sigma_H^{NM}[k=k_F,\ k_F(\mathbf{r})], \tag{133}$$

where $\Sigma_H(\mathbf{r})$ and $\Sigma_H^{NM}(k,\ k_F)$ are the Hartree self-energies of the finite system and infinite matter, both calculated with the effective Lagrangian of Ref. 126.

3) It was assumed that

$$\Delta\Sigma=\Sigma_{HF}(\mathbf{r},\mathbf{r}')-\Sigma_{HF}^{NM}\left[k=k_F,\ k_F\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)\right], \tag{134}$$

where the quantities on the right-hand side have the same definitions as when the approximation (2) is used, but they now include both Hartree and Fock contributions.

The results obtained in the framework of these approximations[125] are presented in Table XI and in Fig. 8. Despite the presence of a certain dependence of these results on the approximations that were used, they are promising and indicate that DBHF theory can be used to describe finite nuclei without phenomenological refitting of the relativistic G matrix, which must be done in the nonrelativistic G-matrix approach.[127,128] The reason for this appears to be that the Dirac–Brueckner approach already includes some three-particle effects, and this leads to closer

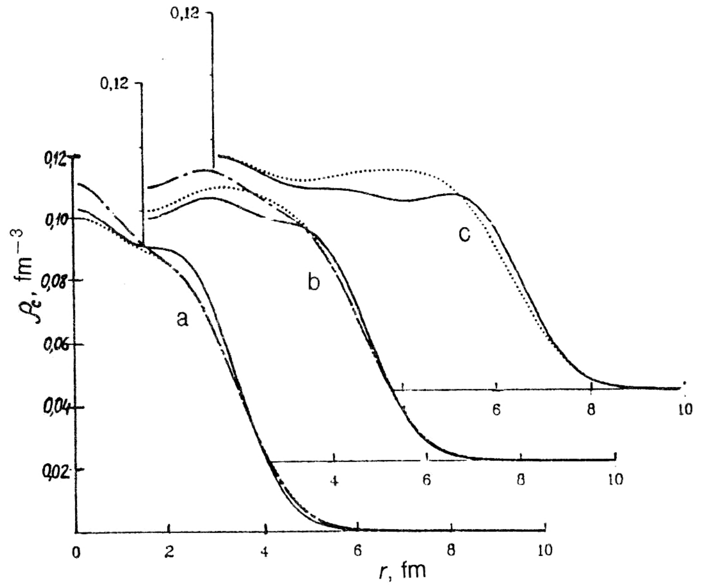

FIG. 8. Calculated charge densities in ^{40}Ca (a), ^{90}Zr (b), and ^{208}Pb (c). The different curves correspond to the approximation: *1*) continuous curve, *2*) dotted curve, *3*) chain curve.

proximity of the calculated and empirical saturation points, whereas in the nonrelativistic G matrix these effects are absent.

7. NUCLEAR EXCITATIONS IN THE RELATIVISTIC MODEL

The success of the relativistic Hartree and Hartree–Fock approximations ensured a good basis for the study of nuclear excitations in relativistic models. As usual, elementary excitations occur because of the displacement of nucleons from occupied states to unoccupied states, while the random-phase approximation (RPA) provides a method for constructing collective states, which can arise as a result of the residual particle–hole interaction.

One of the important points is to establish the possibility of correctly describing in the framework of the relativistic model giant resonances, which are a well-known characteristic realization of nuclear dynamics. It was assumed that because of the large compression modulus the breathing mode would be predicted in the (σ, ω) model at a too high energy. Estimates based on scaling methods also indicate that the isoscalar monopole and quadrupole resonances must lie too high.[129] In what follows, we shall see, however, that these scaling energies are higher than the RPA energies. In this section, we shall dwell on the isoscalar giant resonances because a large proportion of the studies in this field has been made in the framework of the (σ, ω) model, i.e., in this model only the isoscalar particle–hole interaction is described. The calculations of giant resonances that will be considered below do not include the vacuum-polarization effects, which may play an important part. At the present time, a renormalization procedure[57] has been carried out only for infinite nuclear matter[130,131] and has been adapted for finite systems in the local-density approximation. We shall see below some manifestations of the vacuum-polarization effects for the example of the response functions for electron scattering.

We shall discuss briefly the formalism of the relativistic RPA, following Ref. 132. In the Dirac–Hartree approximation, the single-particle spectrum is obtained by solving the Dirac equation

$$[\gamma_0 E_i + i\boldsymbol{\gamma}\cdot\nabla - (M+\Sigma_S) - \gamma^\mu \Sigma_\mu] h_i(\mathbf{x}) = 0, \qquad (135)$$

where (E_i, h_i) are the energy and wave function of the single-particle state i; $\Sigma_S(\mathbf{x})$ and $\Sigma_\mu(\mathbf{x})$ are the self-consistent scalar and vector Hartree potentials generated by the scalar (σ) meson and vector (ω and ρ) mesons, respectively. Using the coupling constants, for example, of Ref. 66, we have at the center of the nucleus $\Sigma_S \cong -400$ MeV and $\Sigma_0 \cong 350$ MeV. We shall distinguish by means of the notation occupied Fermi states ($i\equiv a$, $h_i \equiv f_a$, $E_i \equiv E_a > 0$), unoccupied Fermi states ($i \equiv A$, $h_i \equiv f_A$, $E_i \equiv E_A > 0$), and states of the Dirac sea ($i \equiv \alpha$, $h_i \equiv g_\alpha$, $E_i \equiv E_\alpha < 0$). The single-particle Hartree propagator G_H can be decomposed into Feynman (G_F) and density-dependent $(G_{\mathfrak{D}})$ components:

$$G_H(\mathbf{x},\ \mathbf{x}';\ E) + G_F = G_{\mathfrak{D}}, \qquad (136)$$

where the Feynman and density-dependent propagators are expressed as follows:

$$G_F(\mathbf{x},\ \mathbf{x}';\ E) = \sum_{i=a,A} \frac{f_i(\mathbf{x})\bar{f}_i(\mathbf{x}')}{E-E_i+i\eta} + \sum_\alpha \frac{g_\alpha(\mathbf{x})\bar{g}_\alpha(\mathbf{x}')}{E-E_\alpha-i\eta},$$

$$G_{\mathfrak{D}}(\mathbf{x},\ \mathbf{x}';\ E) = \sum_a f_a(\mathbf{x})\bar{f}_a(\mathbf{x}')\left[\frac{1}{E-E_a-i\eta} - \frac{1}{E-E_a+i\eta}\right]. \qquad (137)$$

If Γ_P and Γ_Q are arbitrary 4×4 matrices, then the uncorrelated particle–hole polarization operator $\Pi^{(0)}$ in coordinate space has the form

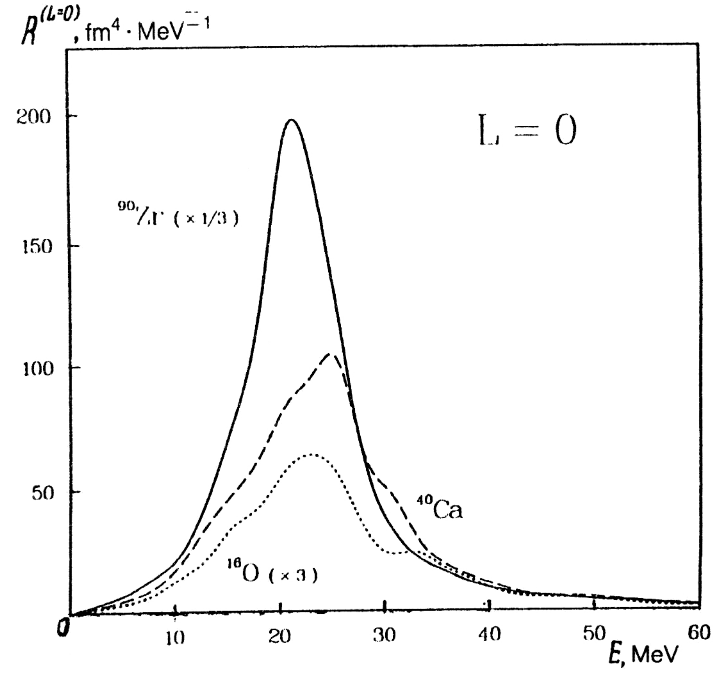

FIG. 9. Monopole response functions $R^{(L=0)}$ as functions of the excitation energy E.

$$\Pi^{(0)}(\Gamma_P, \Gamma_Q; \mathbf{x}, \mathbf{x}'; E) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} dE' \cdot \mathrm{Tr}[\Gamma_P G_H(\mathbf{x}, \mathbf{x}'; E+E') \times \Gamma_Q G_H(\mathbf{x}', \mathbf{x}; E')]. \tag{138}$$

The role of the residual particle–hole interaction reduces to the appearance of a correlated polarization operator Π. If we sum the subclass of ring diagrams, then we obtain the RPA polarization operator. For finite nuclei, it is convenient to use a multipole expansion of Π, since each multipole satisfies a separate one-dimensional integral equation. In the momentum space, we have the following RPA equation for multipole L:

$$\Pi_L(\Gamma_P, \Gamma_Q; k,k'; E) = \Pi_L^{(0)}(\Gamma_P, \Gamma_Q; k, k'; E) + \sum_{a=0}^{4} \int k''^2 dk'' \Pi_L^{(0)} \times (\Gamma_P, \Gamma_a; k, k''; E)\chi_a(k'', E) \times \Pi_L(\Gamma^a, \Gamma_Q; k'', k'; E), \tag{139}$$

where $\Pi_L^{(0)}$ is the Lth multipole of the Fourier expansion for (138), the indices $a=0, 1, 2, 3$ correspond to the ω meson, and $a=4$ to the σ meson. The meson propagators are determined by the expression

$$\chi_a(k, E) = \begin{cases} -\dfrac{1}{(2\pi)^3}\dfrac{1}{E^2-k^2-m_\omega^2+i\eta}, & \text{if } a=0, 1, 2, 3 \\ \dfrac{1}{(2\pi)^3}\dfrac{1}{E^2-k^2-m_\sigma^2+i\eta}, & \text{if } a=4. \end{cases} \tag{140}$$

In Eq. (139), the operators Γ_a have the form

$$\Gamma_a = \begin{cases} -g_\omega \gamma_\mu, & \text{if } a=0, 1, 2, 3 \\ g_\sigma, & \text{if } a=4. \end{cases} \tag{141}$$

If we calculate the uncorrelated polarization operator (138) with G_H with allowance for (136) and (137), then obviously the result will be infinite. The divergence is related to the sum in (138) over all terms of the type $G_F G_F$, which correspond to excitations associated with transition of a nucleon from a state of the Dirac sea to an unoccupied state. In principle, the divergence can be eliminated by means of a renormalization procedure by adding to the Lagrangian counterterms, which lead to a finite result. In practice, this cumbersome procedure has been carried out only for nuclear matter. In the spirit of an effective theory, one can truncate the configuration space by introducing an energy cutoff for the single-particle states (for both positive and negative energies) included in the calculations and investigate the sensitivity of the results to the truncation of the space.[133]

This computational scheme was used in Refs. 132 and 134 to investigate giant resonances. In this region of energies, it is possible to ignore the E dependence of the meson propagators $\chi_a(k, E)$ and use their static limit. In this case, the result (139) is equivalent to the well-known method of solution of the RPA by diagonalization of an energy-independent secular matrix. We discuss some of the typical results obtained in Ref. 132. In that study, the authors did not consider divergence problems; they took into account the contribution of the states of the Dirac sea to the Hartree propagator (136) and the polarization operator (138), i.e., they neglected these states in calculations of the Hartree self-energies Σ. The distributions of the strengths of the multipoles are shown in Fig. 9 for the mode $L=0$ and in Figs. 10 and 11 for $L=2$.

For the isoscalar monopole mode, the model predicts a

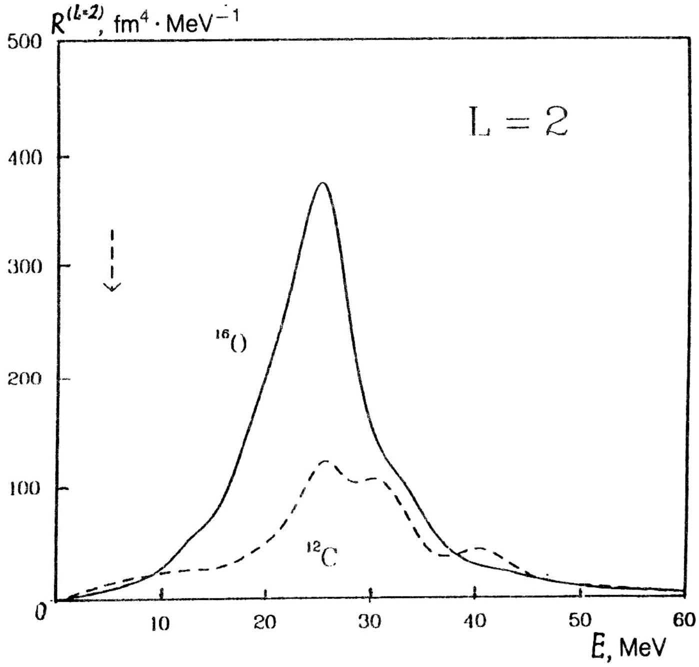

FIG. 10. Quadrupole response functions $R^{(L=2)}$ for the ^{12}C and ^{16}O nuclei. The arrow indicates the position of the low-lying bound state in ^{12}C.

collective resonance for medium nuclei, whereas in light nuclei up to ^{40}Ca the distribution of the strengths is very broad. The calculated energies are higher than the experimental ones. For example, in the RPA the peak energy is 21.5 MeV for ^{90}Zr; this value must be compared with the experimental value 18 MeV. However, in this case the discrepancy is less than the estimate made by Nishizaki *et al.*[129] for the same relativistic model using the scaling approximation.

They established the dependence $160\cdot A^{-1/3}$ MeV for the monopole mode, which gives about 35 MeV for ^{90}Zr. The large difference between the predictions of the two calculations indicates that the compression modulus K does not completely determine the monopole energies on account of the fact that surface effects play an important part.

For the isoscalar quadrupole mode, the scaling results of Ref. 129 agree better with the energies obtained in the RPA. The scaling method gives $E\sim 88A^{-1/3}$ MeV, whereas the RPA calculations lead to peak energies of $\sim 70A^{-1/3}$ MeV for nuclei from ^{16}O to ^{90}Zr. The RPA energies are somewhat higher than the experimental ones, which are determined by the law $60A^{-1/3}$ MeV. To reduce the quadrupole energies, we need a larger value of the effective mass $\mathfrak{M}$ than the value $\mathfrak{M}/M=0.54$ that corresponds to the present calculations.

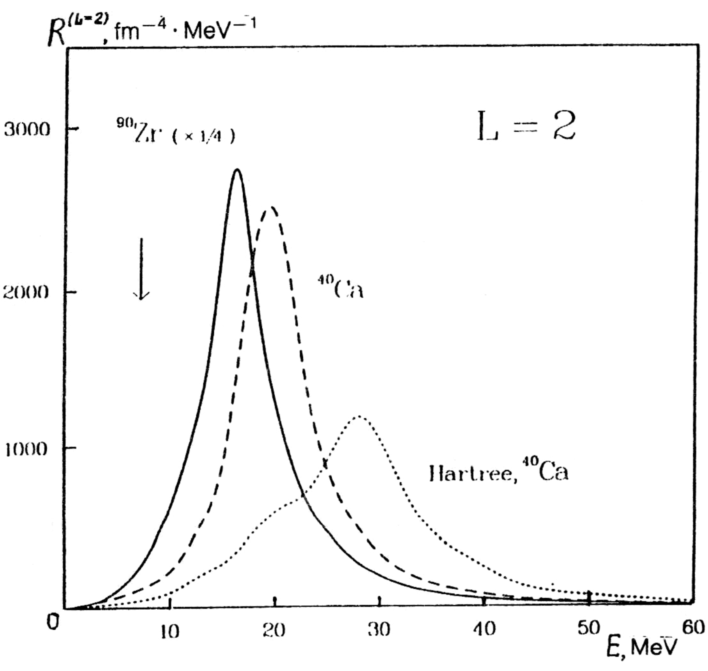

FIG. 11. The same as in Fig. 10, for ^{40}Ca and ^{90}Zr. The dotted curve shows the Hartree response in ^{40}Ca. The arrow corresponds to the position of the low-lying state in ^{90}Zr.

The present RPA calculations do not give information on either the emission widths or the spreading widths. However, they include effects of Landau damping. These effects are larger for the modes with $L=0$ than for the modes with $L=2$. For example, in ^{90}Zr, in which both modes are manifested (as pronounced giant resonances), the Landau width is 4 MeV for the monopole resonance and only 1 MeV for the quadrupole resonance.

Another possibility of making relativistic calculations in the RPA without divergences was proposed by Suzuki *et al.* in Ref. 135, in which a polarization operator $\Pi^{(0)}$ without allowance for the contribution $G_F G_F$ was used. In this case, some effects of the states of the Dirac sea are taken into account through the terms $G_{\mathfrak{D}} G_F$ and $G_F G_{\mathfrak{D}}$.

The relativistic approach has also been used to study nuclear response functions in the region of a quasielastic peak. These response functions can be measured with a high accuracy in electron-scattering experiments[136,137] up to large momentum transfers (~ 500 MeV/c). In Ref. 138, it was already noted that a relativistic model containing strong scalar and vector potentials leads to a significant decrease of the quasielastic peak compared with a nonrelativistic model with a nuclear potential of Woods–Saxon type. However, in Ref. 138 only the independent-particle model was used. Correlation effects have been investigated by various groups[135,139] using the relativistic RPA. They found that the longitudinal response in the RPA is reduced to an even greater extent in the quasielastic peak compared with the Hartree response, and the remaining discrepancy with experiment does not exceed 20%. Recently, vacuum-polarization effects have also been investigated.[130,131,139] The results show that the corrections for the vacuum fluctuations make it possible to improve the agreement with experiment still further. In a recent paper,[140] Kurasawa and Suzuki showed that allowance for relativistic correlations of the RPA with inclusion of corrections for vacuum fluctuations (renormalized RPA) can solve an ancient problem. This problem is that the Coulomb sum rule measured in electron scattering with large momentum transfer gives a result smaller than the one predicted by the nonrelativistic models. Figure 12 gives the results of Ref. 140 in comparison with the data of various measurements.[136,137,141]

Concluding this section, we mention an interesting suggestion made in Ref. 142 in connection with a very specific feature of the relativistic models. It is the possibility in these models of generating not only low-lying excitations of the traditional particle–hole type (giant resonances) but also high-lying $N\bar{N}$ excitations, where $\bar{N}$ corresponds to a hole in the filled Dirac sea. Of course, this simple picture may be changed because the residual particle–hole interaction may couple the $N\bar{N}$ configurations to each other, generating coherent $N\bar{N}$ states. This question was investigated in Ref. 133, where it was established that both types of $N\bar{N}$ state (uncoupled and coherent) can exist at energies much lower than $2M$.

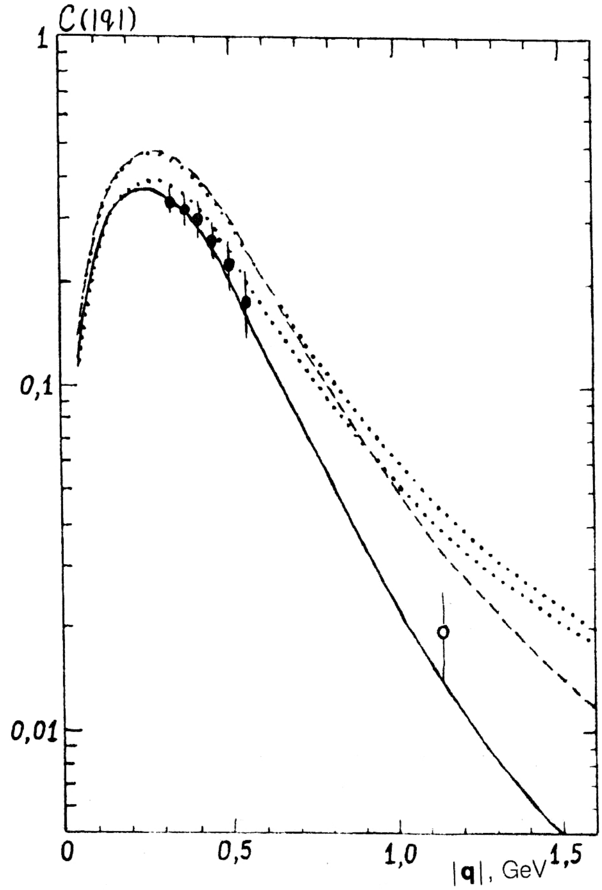

FIG. 12. Value of the Coulomb sum $C(|\mathbf{q}|)$ as a function of the momentum transfer $|\mathbf{q}|$. The black and open circles show the experimental values for ^{56}Fe from Refs. 136 and 141, respectively. The upper and lower dotted curves were obtained in the Hartree approximation and the RPA, respectively, without allowance for the contribution of the sea. The broken curve shows the result for the renormalized Hartree approximation, and the continuous curve gives the result of the renormalized RPA. The curves were calculated for nuclear matter.

CONCLUSIONS

The approach to the description of nuclear properties presented in this review has been intensively developed during the last 20 years. Its achievements and shortcomings are now clear. The approach reproduces a large number of experimental data and leads to a description of nuclear matter and finite nuclei that, at the least, is not inferior to what is achieved by the most sophisticated phenomenological models.

Evaluating the prospects of the approach, we note the following two circumstances.

1. At the present time, it is important to establish not only which of the two approaches in nuclear theory (relativistic or nonrelativistic) is the more economic. Essentially, the answer to this question has already been obtained and presented, in particular, in the present review. It is important to establish which of the two descriptions is more justified from the theoretical point of view. For this reason, the investigations in the immediate future will be concentrated on phenomena in which the relativistic predictions differ radically from those of the traditional scheme of nonrelativistic nucleons. In this respect, there are certain hopes placed, in particular, on the physics of Λ hypernuclei.

2. In recent years, there has been a considerable growth of interest in the investigation of chiral models. There are various realizations of chiral symmetry. It appears very promising to investigate chiral models that can simultaneously reproduce the saturation of nuclear matter and describe the ground states of normal nuclei. At the present time, such calculations do not exist.

Finally we should like to express our deep thanks to I. N. Mikhaĭlov for discussions and helpful advice at various stages of work on this review.

[1)] In Ref. 51, a Lagrangian of the type (5), corresponding to the σ model, is considered. In this case, the Lagrangian does not contain a vector field, and λ and μ in the potential functional $U(\sigma)$ have the values $\lambda=\mu=1$.

[2)] In Ref. 60, the interaction of baryons with pseudoscalar π mesons is treated on the basis of finite-temperature Green's functions.

[3)] In Ref. 27 it was shown that for fixed K and given effective mass $\mathfrak{M}$ (C_V^2 fixed) allowance for vacuum fluctuations increases C_S^2, decreasing at the same time both $\bar{b}$ and $\bar{c}$.

[4)] In this section, the isoscalar vector field V_μ and the isovector vector field b_μ are identified with the ω-meson and ρ-meson fields and denoted by ω_μ and ρ_μ, respectively.

[5)] A difference is that in the relativistic theory the constant α_{LS} in the expression (109) is the strength parameter of the operator of the spin-orbit interaction in the nucleus, whereas the parameter α in the HFS method [see (103)] has no relation to this interaction; it is determined through the parameters t_1 and t_2 of the Skyrme forces [$\alpha=(1/16)(3t_1+5t_2)$].

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Translated by Julian B. Barbour